2 Mensuration

Content

2.1 Conversion of measurements
2.2 Volume of a cube/cuboid
2.3 Area of a triangle

Learning Outcomes

Pupils should be able to:

(a) convert measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa

(b) find one dimension of a cuboid given its volume and other dimensions

(c) find the edge of a cube given its volume

(d) solve up to 2-step word problems involving volume of a cube/cuboid and liquid

(e) use formula to find the area of a triangle

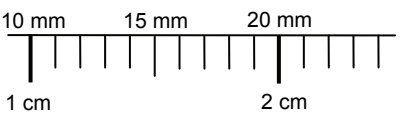
2.1 Conversion of measurements

Introduce

- In everyday life, we come across various measurements. We measure the length of a table, our own weights and volume of water used in our homes.
- Different units are used in different measurements.
- Metre, centimetre, millimetre, feet and inches are used to measure *length*.
- Litre and cubic metre are used to measure *volume*.
- Seconds, minutes, hours, days and years are used to measure *time*.
- In some cases, we take measurement in one unit and convert it into another unit for easy reading.

Conversion of measurements from smaller unit to larger and vice versa

- Let us consider a part of a measuring tape or a ruler which you use to measure length.



- Measure the length of a book. One of you may read 8 cm. Another student may read 80 mm. Both are correct. Both of you have measured the same length in different units.
- In the above figure, 1 cm is divided into 10 parts of a millimetre. One centimetre is larger than one millimetre.

- When converting a larger unit of measurement to a smaller one, we multiply the quantity of the unit by a *conversion factor*.

  Larger unit = quantity of Smaller unit × conversion factor

- In this case, the *conversion factor* for centimetre to millimetre is said to be 10.

- When converting a smaller unit to a larger unit, we divide the quantity of the unit by the *conversion factor*.

  Smaller unit = quantity of Larger unit ÷ conversion factor

- After conversion, the measurement takes on the new unit label.

Length

- Length is measured in kilometres, metres, centimetres, millimetres, feet and inches.

- The kilometres, metres are units of the metric system. Feet and inches are imperial units.

- A length unit conversion table is shown below. The table shows the symbols of the respective units.

1 kilometre (km)	=	1000 metre (m)
1 metre	=	100 centimetre (cm)
1 centimetre	=	10 millimetre (mm)
1 feet (f) (‘)	=	12 inches (“)

Examples

- Convert 1 cm to mm:

  Larger unit = quantity of Smaller unit × conversion factor

  1 cm = 1 × 10 = 10 mm

- Convert 20 cm to mm:

$20\ \text{cm} = 20 \times 10\ \text{mm} = 200\ \text{mm}$

- Convert 1 m to mm:

  $1\ \text{m} = 1 \times 100 = 100\ \text{cm}$

  $100\ \text{cm} = 100 \times 10 = 1000\ \text{mm}$

  Therefore $1\ \text{m} = 1000\ \text{mm}$.

- Convert 1 mm to cm:

  Smaller unit = quantity of Larger unit ÷ conversion factor

  $1\ \text{mm} = (\frac{1}{10})\ \text{cm}$

  It is easier to use the conversion values from the conversion table directly.

☺ When you perform arithmetic operations such as addition, subtraction, multiplication and division between two measurements, they should be in the same units.

☺ When you read a question, notice in which unit the answer should be. It may be easier to convert all the other units into that right from the start.

Mass

- Mass is measured in kilogram and gram, which are metric units.

- We buy vegetables in kilograms and grams. Scientists measure the mass of chemicals in milligrams. Traders measure flour and rice in tons.

- The conversion table provides the relationship between the mass units:

1 ton	=	1000 kilogram (kg)
1 kilogram	=	1000 gram (g)
1 gram	=	1000 milligram (mg)

Example

- A boy's weight is 45 000 g. Convert his weight into kilogram:

  1000 g = 1 kg

  $1 \text{ g} = \frac{1}{1000} \text{ kg}$

  $45\,000 \text{ g} = \frac{45000}{1000} \text{ kg} = 45 \text{ kg}$

  His weight is 45 kg.

Volume

- Volume is measured in litre, millilitre, cubic metre.
- Liquids are usually measured in litres whereas solids are measured in cubic metre.
- The conversion table below shows commonly used volume units:

1 cubic metre (m^3)	=	1000 litre (ℓ)
1 litre	=	1000 millilitre (ml)

Examples

- Convert $\frac{3}{10}\ell$ into ml.

  $1\ \ell = 1000$ ml

  $\frac{3}{10}\ell = (\frac{3}{10} \times 1000)$ ml $= 300$ ml

- A 250 ml bottle is used to remove oil from a 2 ℓ tank. How many scoops does it take to remove all the oil from the tank?

  Convert larger unit into smaller

  Volume of tank $= 2 \times 1000$ ml $= 2000$ ml

  Volume of bottle $= 250$ ml

250 ml oil is fetched in $= 1$ scoop

$\therefore$ 2000 ml oil is fetched in $= \frac{2000}{250} = 8$ scoops

The bottle is used 8 times to remove oil from the tank.

Time

- Time is measured in seconds, minutes, hours, days, months and years.
- Which of these units are used depend on the length of time being measured. The hour unit is used to measure the amount of time we spend in school.
- However, it is more appropriate to use the month unit to measure the time taken for a rose plant to grow.
- The conversion table for time units is given below.

1 Year	=	12 Month
1 Month	=	4 Week
1 Week	=	7 Day
1 Day	=	24 Hour (hr)
1 Hour	=	60 Minute (min)
1 Minute	=	60 Second (s)

Examples

- Convert $\frac{3}{4}$ minutes into seconds.

  Larger unit (min) is converted into smaller unit (s).

  $1 \text{ min} = 60 \text{ s}$

  $\frac{3}{4} \text{ min} = 60 \times \frac{3}{4} \text{ s} = \frac{180}{4} \text{ s} = 45 \text{ s}$

- Let us see how 3 days can be expressed as a fraction of a week.

7 days $= 1$ week

1 day $= \frac{1}{7}$ week

Hence, 3 days $= \frac{1}{7} \times 3$ week $= \frac{3}{7}$ week

Worked Examples

Example 1

Joe uses 20 pieces of 350 mm wooden planks and 2 pieces of 25 cm wooden planks to make a chair. He cut them from a 12 m wood plank.
What is the length of the remaining piece in m?

Solution:

1000 mm = 1 m; 100 cm = 1 m

Total length of 350 mm pieces $= 20 \times 350$ mm $= 7000$ mm
$= 7$ m

Total length of 25 cm pieces $= 2 \times 25$ cm $= 50$ cm
$= \frac{1}{2}$ m

$\therefore$ Total length to be cut for the chair $= (7 + \frac{1}{2})$ m $= 7\frac{1}{2}$ m

Hence the remaining length

$= (12 - 7\frac{1}{2})$ m $= (11\frac{2}{2} - 7\frac{1}{2})$ m $= 4\frac{1}{2}$ m (ans)

Example 2

Ken took $1\frac{1}{2}$ days to travel from Singapore to Canada. He spent around 10 hours in transit in Hong Kong. Find how long he really spent on the flight.

Solution:

Actual time he spent on flight = (No. of days he took to arrive − Transit time)

No. of days he took to arrive in Canada $= 1\frac{1}{2}$ days

24 hours = 1 day

Traveling time in hours $= 1\frac{1}{2} \times 24$ hours $= \frac{3}{2} \times 24$ hours

$= 3 \times 12$ hours $= 36$ hours

$\therefore$ his real flight time $= 36 - 10$ hours

$= 26$ hours (ans)

Example 3

Lina has $2\frac{1}{2}$ m length of a cloth. She needs 200 cm to stitch her skirt. Her child needs 75 cm for a skirt. Can she stitch a skirt for her child with the remaining cloth?

Solution:

Convert $2\frac{1}{2}$ m length to cm $= (2\frac{1}{2} \times 100)$ cm

$= \frac{5}{2} \times 100$ cm $= 250$ cm

The length of remaining cloth = Length of cloth − length of Lina's skirt

$= (250 - 200)$ cm $= 50$ cm

Lina's child needs 75 cm of cloth for her skirt.

Hence Lina cannot stitch a skirt for her child. (ans)

Example 4

Peter bought 1 kg of sugar. His wife took 250 g for her use. Peter packed the remaining sugar into 50 g packets. How many packets would there be?

If 1 kg of sugar is $1.20, what is the cost of one packet?

Solution:

1 kg = 1000 g

The weight of the remaining sugar

= Weight of sugar bought − weight of sugar used

= (1000 − 250) g = 750 g

No. of packets from 50 g of sugar = 1 packet

$\therefore$ No. of packets from 750 g of sugar = $(\frac{1}{50} \times 750)$ packets

= 15 packets (ans)

Price of 1 kg of sugar = \$1.20

Converting units, price of 1000 g of sugar = 120 cents

$\therefore$ 50 g of sugar price $= \frac{120}{1000} \times 50$ cents

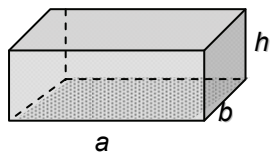
= 6 cents (ans)

2.2 Volume of a cube/cuboid

Find

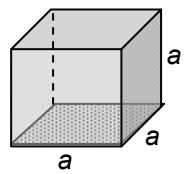
- A **cuboid** is a closed box or a solid object with six rectangular faces. When the six faces of a cuboid are squares then it is said to be a **cube**.

- Volume of a cuboid with length a , breadth b and height h is given by



$$V = a \times b \times h$$

- Volume of a cube with sides of length a is given by,

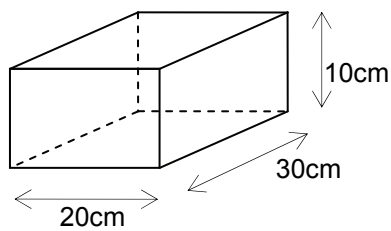


$$V = a \times a \times a$$

Worked Examples

Example 1

Find the volume of the figure as shown.



Solution:

The figure is a cuboid.

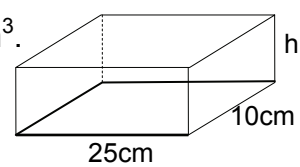
Volume of the cuboid, $V = a \times b \times h$

$= 30 \text{ cm} \times 20 \text{ cm} \times 10 \text{ cm}$

$= 6000 \text{ cm}^3$ (ans)

Example 2

The volume of the cuboid is 3750 cm^3.
Find the height of the cuboid.



Solution:

Let height of the cuboid be h

Volume of the cuboid, $V = a \times b \times h$

$= 25 \text{ cm} \times 10 \text{ cm} \times h$

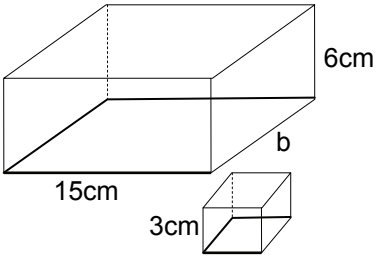
$= 3750 \text{ cm}^3$

$$\text{Height } h = \frac{3750}{25 \times 10} \text{cm} = 15 \text{ cm} \quad \text{(ans)}$$

Example 3

40 numbers of 3 cm height cube can be cut from the cuboid shown in the figure.

Find the width of the cuboid.



Solution:

Let the width of cuboid be b.

The length, width and height are equal for cubes.

Volume of a cube $= 3 \times 3 \times 3 \text{ cm}^3 = 27 \text{ cm}^3$

Hence volume of 40 cubes $= 40 \times 27 \text{ cm}^3 = 1080 \text{ cm}^3$

Volume of cuboid = volume of 40 cubes

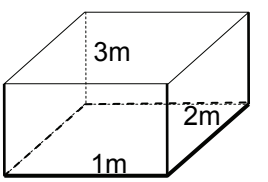
Volume of cuboid $= 15 \text{ cm} \times 6 \text{ cm} \times b$

$15 \text{ cm} \times 6 \text{ cm} \times b = 1080 \text{ cm}^3$

$$\therefore \text{Width } b = \frac{1080}{15 \times 6} \text{ cm} = 12 \text{ cm} \quad \text{(ans)}$$

Example 4

The water tank shown in the figure has to fill with water. A 2 m^3 container is used to pour the water. How many times the container has to be used to fill the tank?



Solution:

Volume of the tank $= 1 \times 2 \times 3 \text{ m}^3$

$= 6 \text{ m}^3$

Volume of the container $= 2 \text{ m}^3$

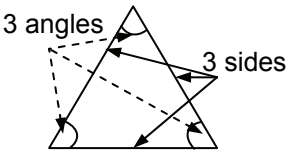
$\therefore$ no. of times the container is used to fill the 6m^3 tank

$= (\frac{1}{2} \times 6)$

$= 3$ times (ans)

2.3 Area of a triangle

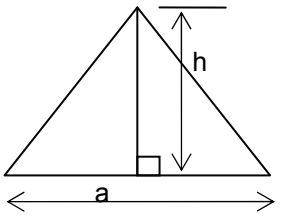
Use



- A triangle is a flat shape with 3 straight sides and 3 angles.

- Area of a triangle, with its base a and height h, is given by

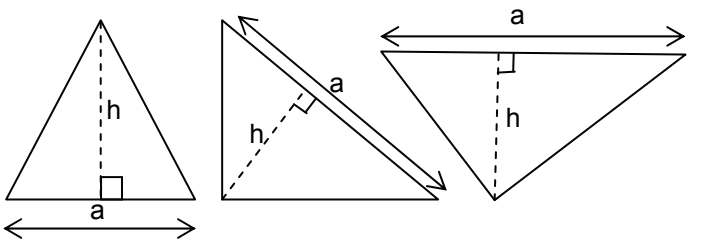
$$A = \frac{1}{2} \times a \times h$$



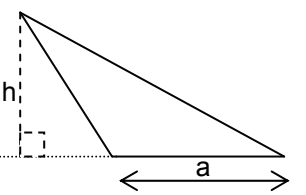
- The base can be any one of the 3 sides of the triangle. The height of the triangle is the shortest distance from the apex, which is opposite to the chosen base.

- The height of the triangle may fall inside or outside the triangle or it can be one of the sides of the triangle, depending on the shape of the triangle.

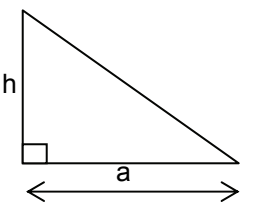
  - Height inside the triangle



  - Height outside the triangle.



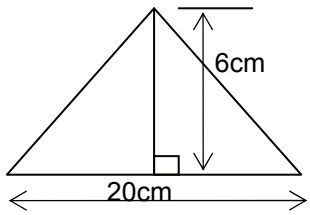
  - Height on one side of the triangle.



Worked Examples

Example 1

Find the area of the triangle.



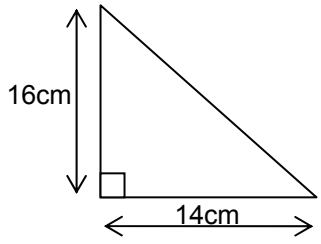
Solution:

$$\text{Area} = \frac{1}{2} \times a \times h$$

$$= (\frac{1}{2} \times 20 \times 6)\ \text{cm}^2 = 60\ \text{cm}^2 \quad \text{(ans)}$$

Example 2

Find the area of the triangle.



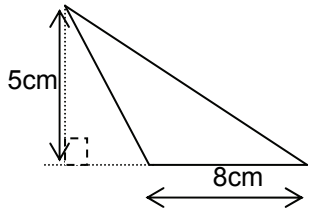
Solution:

$$\text{Area} = \frac{1}{2} \times a \times h$$

$$= (\frac{1}{2} \times 14 \times 16)\ \text{cm}^2 = 112\ \text{cm}^2 \quad \text{(ans)}$$

Example 3

Find the area of the triangle.



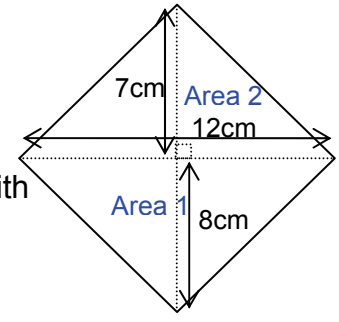
Solution:

$$\text{Area} = \frac{1}{2} \times a \times h$$

$$= (\frac{1}{2} \times 8 \times 5)\ \text{cm}^2 = 20\ \text{cm}^2 \quad \text{(ans)}$$

Example 4

Find the area of the shape.



Solution:

The shape consists of 2 triangles with known length of base and height.

Area of a triangle $= \frac{1}{2} \times a \times h$

$$\text{Area 1} = (\frac{1}{2} \times 12 \times 8)\ \text{cm}^2 = 48\ \text{cm}^2$$

$$\text{Area 2} = (\frac{1}{2} \times 12 \times 7)\ \text{cm}^2 = 42\ \text{cm}^2$$

$$\therefore \text{Area of the figure} = \text{Area 1} + \text{Area 2} = (48 + 42)\ \text{cm}^2$$
$$= 90\ \text{cm}^2 \quad \text{(ans)}$$

☺ Whenever you are asked to find an area of a complicated shape, try dividing it into simple shapes such as square, rectangle and triangle. In this way, it is easy to add up the areas to find the total area of the shape.

Worked Problems

Example 1

Peter has $3\frac{1}{2}$ m of a plastic rope. His mother took $1\frac{2}{5}$ m to tie a parcel. Peter cut the remaining rope into 3 pieces. Find the length of the cut pieces.

Solution:

Length of the remaining rope = Original length − length taken to tie parcel

$$= 3\frac{1}{2} - 1\frac{2}{5} \text{ m} = 2\frac{1}{2} - \frac{2}{5} \text{ m}$$

$$= 2\frac{5}{10} - \frac{4}{10} \text{ m} = 2\frac{1}{10} \text{m}$$

The total length of the 3 pieces $= 2\frac{1}{10}$ m

$\therefore$ Length of one cut piece $= 2\frac{1}{10} \div 3$ m

$$= \frac{21}{10} \times \frac{1}{3} \text{ m} = \frac{7}{10} \text{ m} \quad \text{(ans)}$$

Example 2

Ram's luggage weighed $26\frac{1}{2}$ kg. The airport officer advised him that he was allowed to carry only $22\frac{1}{2}$ kg and have to pay \$25 for each 500 g of extra weight.
Determine how much he has to pay for his extra weight.

Solution:

Extra weight = total luggage weight − allowed weight

$$= 26\frac{1}{2} - 22\frac{1}{2} \text{ kg} = 4 \text{ kg} = 4000 \text{ g}$$

The amount to be paid for 500 g = \$25

$\therefore$ The amount to be paid for 4000 g

$$= \$25 \times \frac{4000\ \text{g}}{500\ \text{g}} = \$200 \quad \text{(ans)}$$

Example 3

An operator can fill 500 numbers of 10 ml bottles with a cough syrup in a day. How long will it take him to fill 700 litres of syrup?

(given 1 month = 30 days)

Solution:

The volume filled in a day $= 500 \times 10$ ml $= 5000$ ml $= 5$l

No. of days taken to fill 5 ℓ syrup = 1 day

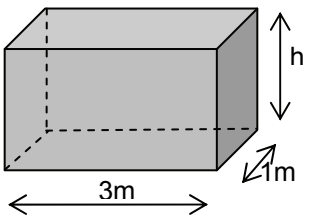
$$\therefore \text{ no. of days to fill 700 } \ell \text{ syrup } = \frac{1}{5} \times 700 \text{ days} = 140 \text{ days}$$

$$= \frac{1}{30} \times 140 \text{ months}$$

$$= 4\frac{2}{3} \text{ months} \quad \text{(ans)}$$

Example 4

A $\frac{1}{25}\text{m}^3$ container was used 100 times to fill the tank full with water. Determine the height of the tank.



Solution:

Let the height is of the tank be *h* m.

The volume filled in 100 times $= \frac{1}{25} \times 100 \text{ m}^3 = 4 \text{ m}^3$

Volume of the tank $= 4 \text{ m}^3$

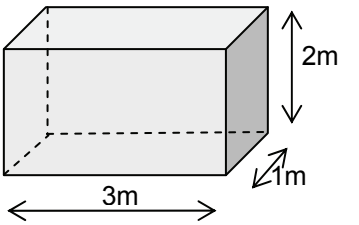
Volume of the tank $= 3 \times 1 \times h \text{ m}^3$

$3 \times 1 \times h \text{ m}^3 = 4 \text{ m}^3$

$$\text{h} = \frac{4}{3}\text{m} = 1\frac{1}{3}\text{m} \quad \text{(ans)}$$

Example 5

Lim needs a 25 m^3 water tank for his home. Determine whether he can use the tank as shown, and if not, find the height that should be extended in order to meet his needs.



Solution:

Volume of the tank $= 3 \times 1 \times 2\ m^3 = 6\ m^3$
Since it is less than 25 m^3 he can not use it.
Let new height of the tank be *h*
Volume of the new tank $= 3 \times 1 \times h\ m^3 = 25\ m^3$

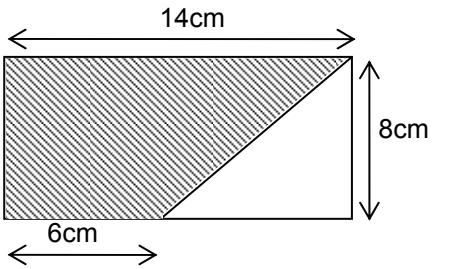
$$\therefore h = \frac{25}{3} m = 8\frac{1}{3} m$$

Hence the height has to be extended by

$$\left(8\frac{1}{3} - 2\right) m = 6\frac{1}{3} m \quad \text{(ans)}$$

Example 6

Find the area of the shaded portion in the figure.



Solution:

Area of shaded figure = area of rectangle − area of unshaded triangle

Area of the rectangle $= 8 \times 14\ cm^2 = 112\ cm^2$

Base of the unshaded triangle $= (14 - 6)\ cm = 8\ cm$

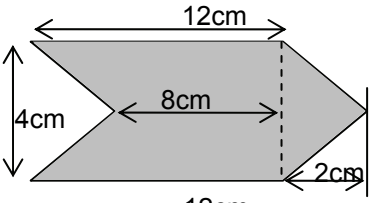
Area of the unshaded triangle $= (\frac{1}{2} \times 8 \times 8) = 32\ cm^2$

$\therefore$ Area of the shaded figure $= (112 - 32)\ cm^2$

$= 80\ cm^2$ (ans)

Example 7

Find the area of the shape.

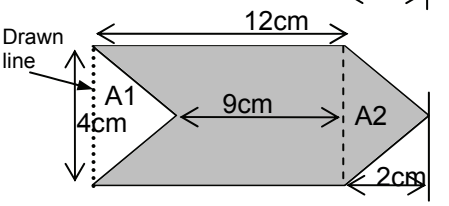


Solution:

Let us draw a line which can make the calculation easier.

Let area of unshaded triangle = A1

Let area of shaded triangle = A2



Area of the shaded figure = area of the rectangle + area A2 − area A1

Area of the rectangle $= 12 \times 4\ \text{cm}^2 = 48\ \text{cm}^2$

Area A2 $= (\frac{1}{2} \times 4 \times 2)\ \text{cm}^2 = 4\ \text{cm}^2$

Area A1 $= \frac{1}{2} \times 4 \times (12-9)\ \text{cm}^2 = (\frac{1}{2} \times 4 \times 3)\ \text{cm}^2$

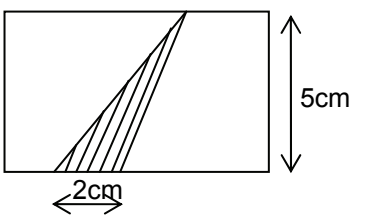
$= 6\ \text{cm}^2$

$\therefore$ Area of the shaded figure $= (48 + 4 - 6)\ \text{cm}^2$

$= 46\ \text{cm}^2$ (ans)

Example 8

Find the area of the shaded section in the figure.



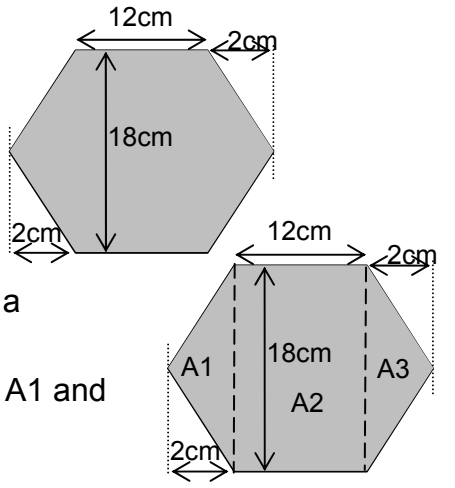
Solution:

The shaded section is a triangle where its height falls outside the triangle.

Area of the shaded triangle $= (\frac{1}{2} \times 2 \times 5)\ \text{cm}^2 = 5\ \text{cm}^2$ (ans)

Example 9

Find the area of the shape.



Solution:

The given figure can be divided into two triangles and a rectangle.

Areas of the two triangles are A1 and A3.

Area of the rectangle is A2.

Area of the shape $= A1 + A2 + A3$

Area A1 $= (\frac{1}{2} \times 18 \times 2)\ \text{cm}^2 = 18\ \text{cm}^2$

Area A2 $= 12 \times 18\ \text{cm}^2 = 216\ \text{cm}^2$

Area A3 $= (\frac{1}{2} \times 18 \times 2)\ \text{cm}^2 = 18\ \text{cm}^2$

Area of the shape $= (18 + 216 + 18)\ \text{cm}^2$

$= 252\ \text{cm}^2$ (ans)