

# What is this book about?

Many currently published books pretend that they are the answer to all the student's woes. They try to prepare the students by trying to teach beyond-syllabus topics and give over-difficult question-types which the student will NEVER encounter in the actual examination. This not only kills the zeal of the student and ABSOLUTELY eats up his study time, leaving him/her mostly unenlightened.

To make up for this lack, we have prepared the **complete guide**. It is **exact syllabus** and complete by itself in every aspect and **every past-years exam questions are presented** within the topical study. There is no need to look for other text material beyond this book.

**You will find our approach in the book refreshing and it ABSOLUTELY saves time and will never, ever over-prepare a student. This book definitely provides an easy path towards the final A\* distinction grade. So why not?**

version 2.12

Some note-worthy latest recorded *question-types*:

- **[2015.p2.10] [Correlation & Regression Section]**

**[Teachers' Comment]** It was expected for candidates to give answer up to 4 decimal places. The final answer mark was not awarded for less accurate answer. Some candidates still did not know how to use the graphic calculator correctly and hence, lead to wrong answer such as wrong values inputed.

- **[2013p1.9(ii)&(iii), 2015p1.1(ii)] [Summation of Series Section]**

**[Teachers' Comment]** Most candidates were unable to give the correct deduction for the second part of the question.

- **[2014p2.4(a)] [Complex Number Section]**

**[Teachers' Comment]** Most candidates found the question requiring difficult algebra, hence, caused candidates having reduced credit, through carelessness.

- **[2012.p2.6] [Sampling Section]**

**[Teachers' Comment]** This question part was not well-answered as the hypothesis test was broken up into various parts.

☺ **Teachers' Comment in 2015 (Differentiation Section)**

This was very badly answered, with the majority of candidates seemingly having not experienced this type of question. A small proportion of the candidates were able to use the identities  $\sin^2 \theta + \cos^2 \theta = 1$  and

$$\frac{\cos \theta}{\sin \theta} = \tan \theta \text{ to solve initial parts but very few realised that}$$

$\tan\left(\frac{1}{2}\pi - \theta\right)$  was equal to the reciprocal of  $\tan \theta$ .

☺ **Teachers' Comment in 2015 (Differentiation Section)**

While many candidates were able to differentiate the given equation correctly, many found the solution of the resulting quadratic in  $e^x$  equated to zero problematic. For those that did solve the resulting equation correctly, very few chose to give an exact form as required choosing instead to give a rounded decimal answer which was then used when determining the nature of the stationary points. This use of a rounded decimal answer was only penalised once.



☺ **Teachers' Comment in 2015 (Summation of Series Section)**

Very few candidates attempted this part of the question and of those did, most chose to ignore the word 'Hence' and so attempted spurious and unsuccessful approaches. Candidates should be reminded that the word 'Hence' implies that work done in the previous part of the question is to be made use of.

☺ **Teachers' Comment in 2015 (Permutations & Combinations Section)**

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

☺ **Teachers' Comment in 2015 (Binomial & Poisson Distributions)**

This paper enabled many candidates to demonstrate their sound knowledge of the syllabus. There were also many candidates however who appeared to recognise the words like normal distribution, binomial distribution, arrangements, mean and standard deviation, but had not studied them in any depth and so were unable to complete the questions successfully. Some calculator work was inaccurate; many candidates lost up to 5 marks because they wrote the correct unsimplified expressions but then appeared to press the wrong buttons on their calculators and thus lost the final accuracy mark.





# Foreword

***“Give a man a fish – and you feed him for a day; teach him to fish – and you feed him for a lifetime.”***

— author unknown

**Knowledge is power. Total knowledge, acquired through accurate and completely true education, is absolute power.** But, there are many publications out there that provide **false information** and it may be difficult for students to separate **accurate education** from this *false information*. Once this false information is learnt, it is very hard for him to eradicate the mistakes; much like learning how to hold the tennis racket without a proper certified tennis coach. Initially, it is not obvious, but when the stress and tempo of the game increases, the student may just damage his elbow and be ousted from the game permanently.

Many of our publications are also meant to double–up as **tradebooks** for the **teachers**; as such these books have to fulfill a much–higher proviso – to assist in the *accurate impartation of true knowledge* to students. Our extremely rigorous publishing control allows us to be confident in that the **reliable truth** the student would ultimately receive.

Every one of the **guide** series is a *fisherman* book – they teach *concepts* that will equip the student for life. Each *concept* is thoroughly explained and further illustrated by *worked examples* and *problems*.

The **critical guide** fundamental series ❶ is best suited for beginners. Although these *guides* are complete on their own, we strongly encourage students to progress to more *extensive guides* as they become more familiar with the subject matter. It is not unusual for some schools to teach beyond the examination (exam) syllabus, to broaden their students’ minds. For this, students are best advised to immediately use the **complete guide** extensive series ❶.

Students with *extreme intelligence* might find the *guides* a little too cumbersome, therefore they should use the *fisherman fast–learner* books, *i.e.*, books that prepare students sitting for the actual exams soon. These books provide the **critical study notes** ❷ for every single exam concept.

Mastering the use of the fishing rod, line, sinker, hook and bait is not enough; one still needs to know the *encyclopedia of fish*, *i.e.*, the **challenging learn–by–example** (LBE) fundamental series ❸. There would be many kinds of fish; success for each may require a subtle but significantly different technique – for example, capturing a shark definitely demands a vastly different technique from that of catching a salmon. In the same way, one must get familiar with the various types of exam question to build confidence. The *challenging LBE* series provides training up to ordinary exam–standard. Again, for students going beyond the norm, the **demanding LBE** extensive series ❸ should be the choice of weapon.

Lastly, one should not neglect the fundamental *work–out* with our ever-popular **challenging drill questions** (with answer keys only) ❹ and **solutions** ❺. Again for students with *extreme intelligence*, do complement learning with our **extreme drill questions** (with answer keys only) ❻ and **solutions** ❼, an all–embracing extensive *work–out* series.

Every year, popular “past-years exam questions” books are usually printed without accompanying accurate solutions, mark schemes and appropriate examiners’ requirements. To make up for this lack, please use our **complete yearly solutions** series ❸ (there are no questions, only solutions). Due to popular demand, we have reinforced some of these commonly–asked–exam–question – types as **challenging practice questions** ❹ and **solutions** ❹. Just before taking the actual exams, one can and should attempt our **challenging examination questions** (intentionally published without answer keys) ❹ and **solutions** (with mark schemes & exam reports) ❿.

Continuous updates and errata are available at <[www.yellowreef.com](http://www.yellowreef.com)> .

***“Using the right tool at each appropriate stage of exam preparation saves time. No point trying to pretend that a simple screwdriver would suffice at all situations.”***

*"If I have seen yonder, it is by standing on the shoulders of giants." – Sir Isaac Newton*

*"The actual journey of inspiration only begins when one takes the first step towards learning humbly from the masters." – paraphrased by the authors*

*"The human form factor remains unchanged, so learn it, love it and master it and you lord over all things." – Chris Hughes*

*"Look at the examples around you, therefore, never chase after money. If truth be told, it is one's passion that gets the ultimate prize. Derive first the pleasure of creating things that are untouchable by all principalities." – Thomas Bond*

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**Motto: Veni video cognito ergo sum**

**Motto in English Literal: I come, I see, I think, therefore I am.**

**Non-literal:** I **come** to this place (by my own free will), I **see** what's going on (learn from the teachers), I **think** of my current position and contemplate my future, then I become the person I want to **be** (my best self).

It starts when the student freely chooses to **come** to this place for the want of knowledge. This free will is important to the student. Without the free will, the student will find the subsequent knowledge acquisition arduous and tough.

From this place, the student will choose his giant teachers to acquire in-depth knowledge and **see** the true paths of these giants ("*learning from the giants*" by Isaac Newton).

After knowing the ways of the giants, the student will **think** about his current position and at the same time, choose and decide his future weapon-of-choice. This weapon is his answer and challenge to the world. The student will proceed to create and hone this ultimate weapon.

The student will then become the giant he wants to be and **be** his best self ever in perfect harmony with the galaxy.



The **University of Cambridge** (informally *Cambridge University*, or simply *Cambridge*) is a public research university in Cambridge, England built in 1209. It is the second–oldest university in both the United Kingdom and the English–speaking world, and the seventh–oldest globally. In post–nominals the university's name is abbreviated as **Cantab**, a shortened form of Cantabrigiensis (an adjective derived from Cantabrigia, the Latinised form of Cambridge). Academically *Cambridge* ranks as one of the very top universities in the world. *Cambridge* regularly contends with *Oxford* for first place in UK league tables. In the most recently published ranking of UK universities, published by The Guardian newspaper, *Cambridge* was ranked first.



The **University of London** is a federal university made up of 31 affiliates: 19 separate university institutions, and 12 research institutes. As such, the *University of London* is the largest university in the UK by number of full–time students, with 135,090 campus–based students. The university was first established by a Royal Charter in 1836. Graduates of the *University of London* often use the post-nominal letters '**Lond.**' or, more rarely, '**Londin.**' (both from Londiniensis) after their degree abbreviations. Academically *University of London* remains one of most advanced and top–ranking universities in the world.

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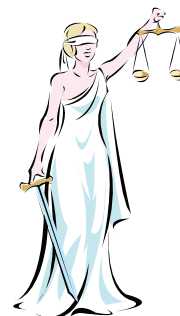
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## BOOK TYPES

- ① *personal journey*
- ① *critical guide*
- ② *critical study notes*
- ③ *challenging learn-by-example*
- ④ *challenging practice questions*
- ⑤ *challenging practice solutions*
- ⑥ *complete yearly solutions*
- ⑧ *complete yearly solutions*
- ⑨ *challenging examination questions*
- ⑩ *challenging examination solutions*
- ① *complete guide*
- ③ *demanding learn-by-example*
- ④ *challenging drill questions*
- ⑤ *challenging drill solutions*
- ⑥ *extreme drill questions*
- ⑦ *extreme drill solutions*

## MARK SCHEME LEGENDS

Marks are of the following four types:

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula.  
  
Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- C** Calculation/Compensation marks are interchangeable with Method marks, but are more specific to marks awarded to calculations
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained.  
  
Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Accuracy mark for a correct result or statement independent of method marks.

## MARK SCHEMES

Every examination has a *mark scheme*. **Mark schemes** are prepared by the **Chief Examiner** and considered, together with the relevant questions, by a panel of subject teachers. The *mark scheme* includes any amendments made at the *standardisation meeting* attended by all examiners and is the scheme which was used by them in the examination. The **standardisation meeting** ensures that the *mark scheme* covers the candidates' responses to questions and that every examiner understands and applies it in the **same correct way**.

## EXAM REPORTS

Every examination has an *examiner's report*. The **Examiners' Report** may refer in general terms to statistical outcomes. Statistical information on candidates' performances in all examination components are provided when results are issued; as well as a review of the performance of candidates in the examinations and detailed analysis of the standards of answering, especially **erroneous understanding and wrong application of relevant concepts**.

## CONTROVERSY

One would have assumed that the aim of the examination is to gauge and qualify candidates according to his/her standard of achievement and competence in the subject matter. Hence, after every examination, the *chief examiner* (internal or external) through the exam board will issue both the *mark scheme* and *exam report* to every school as a guide for teachers and future candidates helping them handle examinations more accurately.

Over the years, with the help of these relevant *mark schemes* and *examiner's reports*, candidates have steadily raised the quality and standard of answers.

The grades were concentrated at the top–end of the spectrum. This has caused problems to the exam boards trying to distinguish the better candidates from the poorer ones.

Instead of implementing a better qualification process, some exam boards chose the easy way out. These exam boards completely removed access to these important *mark schemes* and *examiner's reports* and classified these documents as “**confidential**”, thus, running contrary to the *original spirit of education*.

**We do not believe in this opacity.** We believe in educating students through *total true knowledge*. Hence, in **all** our publications, we have, as far as practicable, included all relevant *mark schemes* and *exam reports* to as many question types as we could possibly get hold of.

We have a duty to ensure

**“Knowledge is power. Total knowledge, acquired through accurate and completely true education, is absolute power.”**

*One simply must know how an examiner marks and what constitutes a complete and correct answer assuring **full credit**.*



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## A token of appreciation

Try as hard as we may for perfection, no matter how careful and repetitive the efforts, errors in our publications are, unfortunately, inevitable; some typographical, factual or calculation oversights still elude us. For that, we hope you could accept our sincere apologies here and would greatly appreciate it if you could kindly point them out to us either through the email [[sales@yellowreef.com](mailto:sales@yellowreef.com)] or website.

In our continuous and relentless pursuit of excellence, we would like to extend a very small token of appreciation to you for these selfless feedbacks (per mistake reported and accepted).

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By this partnership of *collective wisdom*, we truly hope to provide **accurate true education** to users of our publications.

Our management hereby reserves the rights to the compensation aforesaid to all original submissions. Due to the large number of *repeated* submissions, only the first successful original submissions are notified and awarded.

We also would like to take this opportunity to thank the numerous well-wishing contributors and wish all of you many happy returns. Without your support and feedbacks, we wouldn't be able to improve so quickly, accurately and truly. Once again, a big thank you.

version 1.20

## TOTAL EXAM COMPATIBLE

Many notes and guides made available in the market-place are just mere summaries or shortened versions of established text. This would not help the student taking the exams. Our notes and guides are intentionally made "*exam-compatible*", *i.e.*, every exam question can be answered simply by "*cutting-and-pasting*" the required notes or statements of fact from our publications to receive **full credit**.

# Note from the Authors

## Critical ❶ / Complete ❶ Guide

*“A **guide** is a person who leads anyone through **unknown** or **unmapped** country. This includes a guide of the real world (such as someone who conducts travellers and tourists through a place of interest), as well as a person who leads someone to more abstract places (such as to knowledge or wisdom).”*

— Wikipedia

A **textbook** on the other hand is a manual of instruction in any branch of study. *Textbooks* are produced according to the demands of educational institutions (Source: *Wikipedia*).

I have come across many students whose feedbacks were unbelievably similar:

*“Since it is compulsory for me to get the textbook, I feel that there is no further need to get the guide. The guide is no more than regurgitating information from the textbook, I would rather try to learn by studying the textbook first and then follow by practising some of the readily available exercises provided by the teacher. I believe teacher knows best. When the going gets tough, it is usually due to my large class size. I will then seek private tuition instead.”*

Unfortunately, no matter how good the private tutor is, the private tutor is only willing or able to spend, at most 1–2 hours per week per subject coaching the student. This approach is clearly inadequate; else, every single student would have achieved easy distinctions. However, obviously, this is not the current scenario. **Accurate self–education is therefore of paramount importance.**

Having a **competent guide** is extremely important for the development of a student’s competence in the subject matter. The *guide* collates each and every important curriculum in the subject matter and presents it in a professional and logical manner. The aim is to provide **expert guidance** to the student by helping him to achieve competence as fast as possible; much like the *local trekking guide* who will take the unfamiliar climber through the safest and smoothest **uncharted passage** to the mountain top; regardless of his own competence. Although never mentioned in any of the official trekking guides, descending is way more treacherous than ascending. Hence, a competent mountain guide will always prepare a hand–made “walking stick” for his charges’ descend. This way, most of the permanent injuries associated with reckless descending can and will be minimized. Amusingly, contrary to popular belief, ascent is deemed very easy and safe.

To illustrate this important relationship, let’s go through the various aspects of a **competent guide**:

### ❶ Question–answering techniques:

*Textbooks* are not allowed to teach *question–answering techniques*. It is forbidden by the education institution. Education institutions are expected to maintain a holistic approach to the curriculum and to education. **Spoon–feeding is clearly not one of them.**

#### Example

*What is the difference between speed and velocity?*

Most students (and some junior teachers) would have smiled at the simplicity of the question asked and naively suggested the answer to be:

*Speed is a scalar quantity, with only a magnitude defined; while velocity is a vector quantity, with a magnitude and a direction defined.*

The students expect no less than a full credit. But, they are mistaken.

The examiner expects the student to be able to explain the defining difference unique to this case. Otherwise, the question could easily have been between the quantities of say, speed and magnetic field, or between air and building material, or any other two different things. Clearly the examiner meant that speed and velocity are somehow related and



hence, the examiner intends to examine the student's ability to discern its particular difference. The expected answer is therefore:

*Velocity is a vector quantity, with a magnitude and a direction defined; while speed is only the magnitude of the velocity, hence, a scalar quantity.*

Subtle, but different. This answer would have gained full credit.

② **Definitions:**

**Example**

*Define speed?*

Some students would have suggested the answer to be:

*Speed is defined as the change in the distance travelled per unit time.*

Some schools would have given full credit to this answer. But, some top schools would have deemed this answer as 'incorrect'. These top schools expect the students to give the answer as:

*Speed is defined as the change in the distance travelled with respect to time.*

The top schools reasoned that the former answer is a physical formula and not a statement of physical fact, which implies that the statement could not be applied in other planets or different system of base quantities.

A competent guide would have collated all these relevant examiner's reports to ensure the correct presentation of **accurate and true education**, so that the student can confidently score. A *textbook* doesn't perform this role.

③ **Working knowledge:**

**Example**

*Find the inverse of the function*

$$f : x \mapsto \frac{1}{x-1}, \quad x \in \mathbb{R}, \quad x \neq 1.$$

Again, a fair number of local textbooks suggested the following working to the final answer, which many students not being wiser followed:

$$\text{Let } y = f(x) = \frac{1}{x-1},$$

*Rearranging and making x as the subject,*

$$x-1 = \frac{1}{y} \Rightarrow x = \frac{1}{y} + 1$$

*∴ The inverse function is:*

$$f : x \mapsto \frac{1}{x} + 1, \quad x \in \mathbb{R}, \quad x \neq 0. \quad \text{(ans)}$$

The inverse function cannot "miraculously" appear after the statement,  $x = \frac{1}{y} + 1$ . It is

just not possible. **Mathematical deduction must be made on firm mathematical ground.** By being able to deduce the final answer is not the end itself, the approach must be sound and persuasive. Therefore, before granting full credit, the examiner must be convinced that the student knows the exact reasoning behind the derived answer.

The competent guide would have suggested this simple and yet exact approach:

$$\text{Let } y = f^{-1}(x), \text{ the required inverse function.} \quad \text{— ①}$$

*Perform a function operation on y:*

$$f(y) = f[f^{-1}(x)]$$

A property of function: Any function of variable,  $x$ , being operated upon by its own inverse function will yield back the variable,  $x$ :

$$\Rightarrow f(y) = x$$

$$\Rightarrow \frac{1}{y-1} = x \Rightarrow \frac{1}{x} = y-1 \Rightarrow \frac{1}{x} + 1 = y = f^{-1}(x)$$

$\therefore$  From ①, the inverse function is:

$$f : x \mapsto \frac{1}{x} + 1, x \in \mathbb{R}, x \neq 0. \text{ (ans)}$$

#### ④ Facts:

##### Example

Posted on <stomp.com.sg> on 10 Nov 2009

MOE approved Physics textbook teaches my nephew the wrong thing.

A STOMPer's nephew came home crying after his GCE 'O' level Physics exam. He answered a question according to what he studied from the MOE approved textbook, only to find out later that it was wrong.

How can the MOE approved textbooks, which so many students use, carry such errors and ambiguities, questions this STOMPer.

Says this STOMPer:

"My nephew who sat for his GCE 'O' level exam came back crying and showed me his Physics Paper 1 (5058/01) in which Q19 says:

- What is the refractive index of a medium?
- A the ratio of the speed of light in air to the speed of light in the medium.
  - B the ratio of the speed of light in the medium to the speed of light in air.
  - C the ratio of the speed of light in the medium to the speed of light in vacuum.
  - D the ratio of the speed of light in vacuum to the speed of light in the medium.

"According to the internet, which I have checked the answer is 'D'.

My nephew who wrote the answer as 'A' later consulted his teacher after the exam and was told that the answer is 'D'.

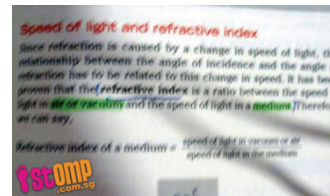
According to his textbook page 231 the authors say 'It has been proven that the refractive index is a ratio between the speed of light in air or vacuum and the speed of light in a medium'.

The cover of the school textbook titled GCE 'O' Level Physics matters by Charles Chew and Chow Siew Foong and there was a seal with the words 'Approved by Ministry of Education for use from 2007 - 2011'

This was the textbook used in many of our secondary schools and endorsed by the MOE.

If the textbook has errors or ambiguities why does MOE recommend that schools use this particular textbook?"

A competent guide would have explained that the determination of any physical standard, such as the refractive index, must be found based on invariant bench-mark(s). Air being







denser or less dense in different parts of earth would not have been a good candidate. At best, it can only be used as a good proxy or approximation.

***“Knowledge is power. Total knowledge, acquired through accurate and completely true education, is absolute power.”***

version 1.07

## Challenging Practice Questions ④ / Solutions ⑤

***“Now, here, you see, it takes all the running you can do, to stay in the same place. If you want to get somewhere else, you must run at least twice as fast as that!”***

– Lewis Carroll (author, *“Alice Through the Looking-Glass”, a sequel of his earlier work, Alice in Wonderland*)

The *Red Queen* is the one who runs hard but never gets anywhere because everything else in the landscape is also running. She tells Alice, “It takes all the running you can do to keep in place!” *Van Valen* used the *Red Queen* as a metaphor for his evolutionary principle – regardless how well a species can adapt to its current environment, it must keep evolving to keep up with its competitors and enemies who are also evolving. Thus, the **“Red Queen” effect**: do nothing and fall behind, or run hard to stay where you are.

**Leigh Van Valen** was one of the founders of the field of Paleo-biology, the combination of research on current life forms with the study of fossils to answer questions about large-scale evolutionary and ecological patterns. He was the famous evolutionary biologist, who coined the **“Red Queen Theory of Evolution”**, *i.e.*, as we improve, our competitors, preys and predators improve too.

Not only available resources for the student limited, **time is also scarce**.

Hence, one’s **natural choice** would therefore be to get the best guidebooks and procure some of the “hardest” and “trickiest” question books to practise. This works extremely well for the preliminary exams; the student scores extremely well. Most schools set their papers with question–types that are deemed the hardest and trickiest. Most students are exam–smart and are extremely proficient at “spotting” the question–types correctly. Most schools feel that after being tested on the most difficult question–types, the student is therefore now better prepared for the actual final external exams. How misleading!

For the actual final external exams, the student noticed that the question–types were a lot easier and demanded very few “acrobatics” but more than 50% of the questions asked were for explanations from the first principles! Something that the school took for granted, since it is so fundamental. It is sad that during this time, being ill–prepared or more accurately wrongly–prepared, some of the most able students (those who scored well in their prelims) leave the exam hall distressed, many have suffered immense mental agony as a result (some even gone mad). Unsurprisingly, many did very badly and tutors were puzzled by this poor performance. Good prelim results apparently did not show a good correlation with actual exam grades. For years, tutors were dumb–founded by this unusual display of diversity and frequently attributed it to “just–in–time” illnesses or sudden mental blocks.

If a student is well–prepared and comfortable about his / her ability, any exam is no more complicated than a common class test or classroom exercise. It should not induce any high mental stress that might cause extreme mood swings and mental discomfort.

Make no mistake about exam stress. The student must be prepared and aware of the question–types being tested in the actual final exam. It is a true battle, one must be clearly aware of the situation. There is absolutely no point scoring a distinction for the prelims while securing a less than ideal grade or even worse, a fail for the actual finals. It is the final exam grade that truly matters.

Therefore, one must practice and be well–versed in the actual question–types for real and most important final external exam.

The *challenging practice questions / solutions* set collates all these **actual question–types** and present them in a logical order to meet this exact need, a **must–have critical resource for the student**.

With all **‘trick’ questions covered** and **complete coverage of similar question–types since 1996**, one could hardly go wrong.

Give yourself a break, even if you have no *time*, practise this series, you can well be on the way to an **easy final distinction**.

version 1.04

## Challenging Drill Questions ④ / Solutions ⑤

***“If I have seen yonder, it is by standing on the shoulders of giants.”***

— Sir Isaac Newton

*If a student is drilled to the hilt on the question–types, wouldn’t the student be only exam–smart and may not be able to handle life or career at a later stage?*

This statement is only half–true when it comes the ultimate grading of students.

If a student is able to do nothing else, except to be drilled well and consequently, score well, then it is true that student may not do well in life after school. This student would be better off just by learning a basic technical skill and working hard for the rest of his / her remaining life. But, we are dealing with very intelligent beings here, the **homo sapiens** species. Scientists had all along estimated that our human brain is 40% too big. Even the lion, the earth’s most ferocious predator, king of all beasts, has a brain that is much smaller (in proportion) than that of the human species.

So, what is the cause of this evolutionary misnomer?

The answer lies in the fact that although we, humans, being right at the top of the food chain and lack natural enemies, yet, we are our own and worst enemy. The constant fear of our *neighbours* and the resulting mental exercise is the main reason for the “grossly magnified” growth of our brains, through natural selection (the **evolution theory**). Hence, it is only natural that humans with more developed neural faculties are better adept at avoiding danger from his *neighbours* resulting in a greatly increased chance and rate of survival.

Recently, scientists have also dwelt in depth on the all important issue of **nature versus nurture**. A Cambridge scientist concluded that the human brain is much more sophisticated than we think it is. Instead of suggesting that we are born more or less capable than our *neighbours*, he suggested that the human brain is born very versatile and that the natural mind is built with adaptors to take advantage of his environment. This explains why a child born of “poor” parents can still ace the exams and do well in life, while a child born of more privileged origins may still fall by the way–side. Or a person with limited education can still build wealth beyond their wildest dreams, at the same time, a child with rich disposition and education, still lost and indecisive about his path. (Here I chose wealth as a measure for success, of course, one can choose other measures, but it should not affect the outcome of case study.)

Since there are so many examples supporting both the necessity and superfluous nature of education, can we conclude that the latter is true?

The answer is obviously no and quite the opposite. The above–mentioned Cambridge scientist concluded that the nurture part of the child is more significant. Thus, the need for education is greater than ever.

Education is about learning what our predecessors have discovered. It would be absolutely inefficient and redundant to deny education and reinvent the wheel. Education for that reason alone, is the shortest path to success. **By knowing and mastering existing knowledge, one now has the luxury of more time to uncover new truths and be better** (*i.e.*, a long–winded way of restating **Newton’s quote**).

This *challenging drill questions / solutions* set is meant for direct use by students preparing for the exams. A sky diver repeatedly jumps to achieve precision and agility. A fighter pilot repeatedly flies his aircraft to co–ordinate his attack sequence and mission completion. A general of the army



frequently conducts paper and ground exercises to prepare for that possible eventuality of war. Yet, no one laughs at their doggedness but instead, commends their fervour.

When it comes to students drilling for exams, these conscientious students have been labelled and teased as nerds and geeks (muggers). It is “uncool”. This is not right. It came as no surprise that Mr Bill Gates, Microsoft, one time the youngest and richest billionaire, once said that “*the nerds and geeks will rule the world*”.

- ❶ Every year, examiners from top schools like to create **new question–types** to test their students. This book collects faithfully these new question–types and presents them in its entirety in a **topical order to facilitate careful rapid mastery** and learning (**with complete answer keys**).
- ❷ No matter how creative the examiners are, there is a physical limit to the number of question–types one can reasonably create. The question–types are limited by the exam syllabus. This book is a **complete and thorough encyclopedia of question–types**. There shall be no surprises.
- ❸ Students are normally quite adept at dealing with *normal* question–types, but the exams more often than not include many “trick” questions. One would notice the more recent exam question–types were involved in advanced “trickery”. “Tricks” are only surprising when they are encountered fresh. If the same “trick” reappears, the “magic” would have worn-off and no longer have any effect on the student. **But, these “trick” question–types must first be encountered.**
- ❹ Due to the time–constraint and vast array of question–types, it may not be reasonable to insist that the student practice every single question presented in the *questions book*, after all these questions tend to be more difficult and hence, more time–consuming. The *solutions book* with **step-by-step solution** to each question, serves as a remedy for this lack.
- ❺ Top schools by far set the most difficult question–types to drill their students. If the student can answer the questions in this *question book*, one can confidently score in every single exams. **The tendency towards carelessness is also greatly reduced.**

version 1.03

# Levelling the playing field

A **level playing field** is a concept about fairness, not that each player has an equal chance to succeed, but that they all play by the same set of rules.

Not every person is born with a silver spoon in his mouth and access to the same resources. There is, therefore, no such thing as a *level playing field* to begin with. Even so, it is not a peccadillo to be born poor.

Although wealth is not an absolute measure of one's happiness and self-worth, it nevertheless provides one with a sense of well-being.

If one is unskilled, one has no choice but to take on only lowly paying jobs which require little or no job-specific knowledge.

In order to improve one's standing in the society, one has to be schooled in both knowledge and skill. In the broadest sense of the word, **education** refers to any act or process of imparting or acquiring general knowledge and developing the powers of reasoning and judgment that has a formative effect on the mind, character, or physical ability of an individual, and generally of preparing oneself or others intellectually for mature life. It is usually transmitted through schools, colleges and universities.

With *education*, one can choose to be a chef, a dancer, an artiste, a lawyer, an engineer, a doctor, an accountant or even a banker. Even though one might have come from a lowly background, one can still advance comfortably to higher rungs of the society and improve one's life.

Despite this, if one still has the passion to be the best street-cleaner or dish-washer in town, why not? Indeed one can truly choose his / her own path.

Hence, *education* is about having one's own choice and definitely one's ultimate field leveller.

**But, the process of true education is not cheap.**

We are prepared to assist in whatever ways we can to provide this field leveller. Any students that are currently under any *financial assistance schemes* or deem unable to procure any of our publications to improve their grades or if you are a compassionate teacher or concerned parent involved with these students, please do get in touch with us at <[eBooks@yellowreef.com](mailto:eBooks@yellowreef.com)> on the number of **free copies** you need.

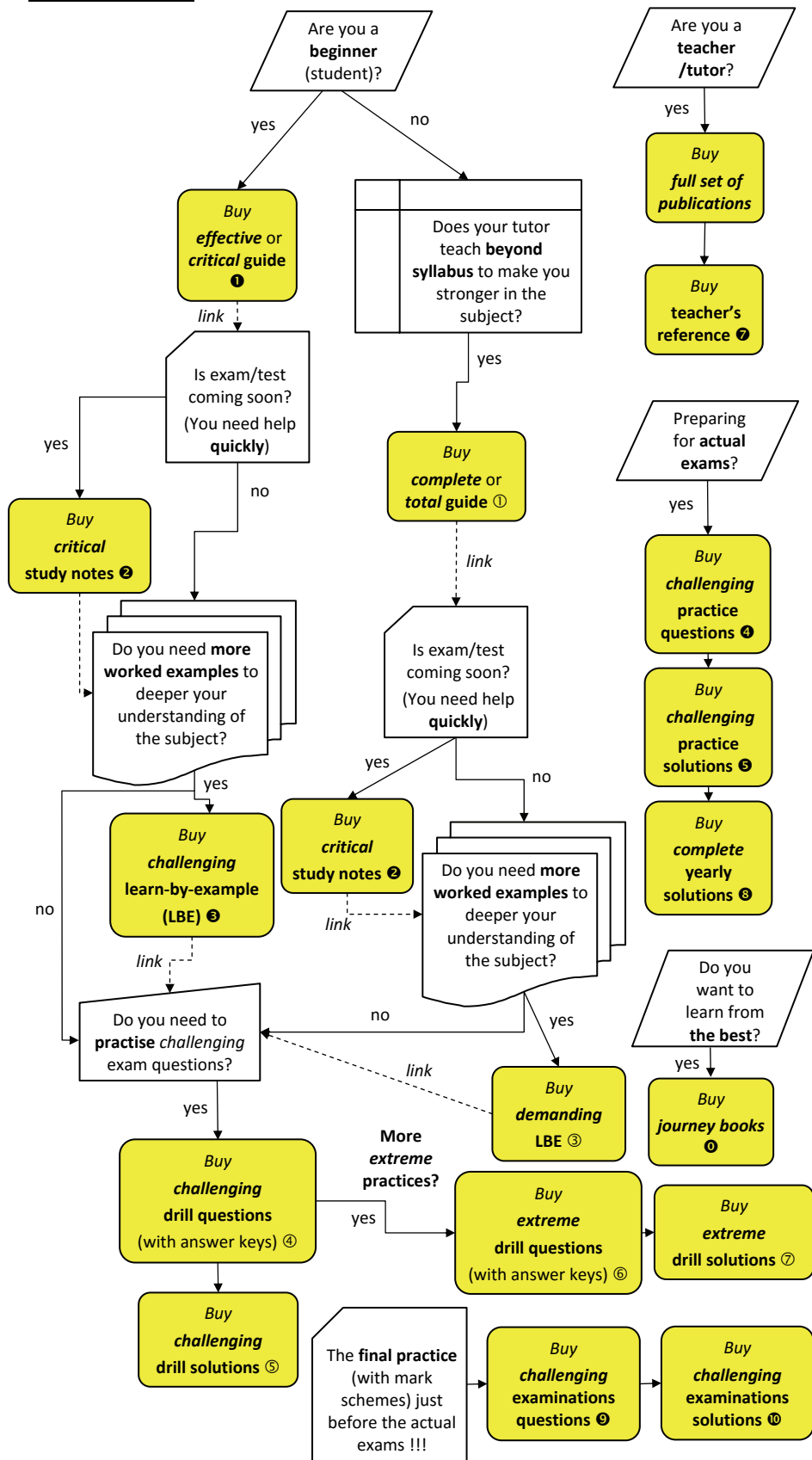
For those of you who are not eligible for this help, but still want some form of help, please visit <[www.yellowreef.com](http://www.yellowreef.com)> and look under the <*books*> or <*solutions*> sections for some free goodies.

Here, may we wish all talents, big and small, be allowed to blossom to their fullest extent.

version 1.11



# ROAD MAP



version 2.07

*igcse or ordinary level*  
(S1 – S3 or S4)

*cambridge*  
*advanced level*  
or *DSE*

<b>critical guide ①</b>	An ample <i>effective</i> guide for students starting out at junior Secondary 1 and ends at Secondary 3 or 4 or 5.	An ample <i>effective</i> guide for students starting out at Secondary 4 or lower Form 6 or JC year 1 and ends at Secondary 6 DSE or upper Form 6 or JC year 2 or 3.
<b>complete guide ①</b>	A total deliberation of extensive concepts taught in <i>top schools</i> .	A total deliberation of extensive concepts taught in <i>top colleges</i> .
<b>critical study notes ②</b>	An intense and sharp delivery of concepts. The structure and organization are direct verbatim imports from <i>top schools</i> .	An intense and sharp delivery of concepts. The structure and organization are direct verbatim imports from <i>top colleges</i> .
<b>challenging learn-by-example (LBE) ③</b>	The MCQs / structured question-types were taken from actual exams. The deliberations were from easy to hard. (topical)	The MCQs / structured question-types were taken from actual exams. The deliberations were from easy to hard. (topical)
<b>demanding LBE ③</b>	The MCQs / structured question-types were taken from actual exams worldwide. The deliberations were from easy to hard. (topical)	The MCQs / structured question-types were taken from actual exams worldwide. The deliberations were from easy to hard. (topical)
<b>challenging practice questions ④</b> (with answer keys) <b>/ solutions ⑤</b>	Actual exam questions arranged in a topical order. Its <i>comprehensive</i> solutions, with mark schemes.	Actual exam questions arranged in a topical order. Its <i>comprehensive</i> solutions, with mark schemes.
<b>challenging drill questions ④</b> (with answer keys) <b>/ solutions ⑤</b>	For practice. The MCQs / structured question-types were taken from actual preliminary exams of <i>top schools</i> . (topical) plus <i>comprehensive</i> solutions.	For practice. The MCQs / structured question-types were taken from actual preliminary exams of <i>top colleges</i> . (topical) plus <i>comprehensive</i> solutions.
<b>extreme drill questions ⑥</b> (with answer keys) <b>/ solutions ⑦</b>	For practice. The MCQs / structured question-types with solutions were taken from actual exams. (topical)	For practice. The MCQs / structured question-types with solutions were taken from actual exams. (topical)
<b>teacher's reference ⑦</b>	A complete and comprehensive reference for teachers.	A complete and comprehensive reference for teachers.
<b>complete yearly solutions ⑧</b>	The solutions to actual exams arranged in a yearly format. (only solutions, no questions; please purchase your own copy of the questions)	The solutions to actual exams arranged in a yearly format. (only solutions, no questions; please purchase your own copy of the questions)
<b>challenging examinations questions ⑨</b> (with answer keys) <b>/ solutions ⑩</b>	Accurate exam questions arranged in paper order. The answer key was deliberately not provided as it intended to simulate the actual exam condition. plus <i>comprehensive</i> solutions, with mark schemes.	Accurate exam questions arranged in paper order. The answer key was deliberately not provided as it intended to simulate the actual exam condition. plus <i>comprehensive</i> solutions, with mark schemes.
<b>journey books ⑩</b>		For in-depth learning.

version 1.90