18

alternating current

18.1 Characteristics of alternating currents

18.2 The transformer

18.3 Rectification with a diode

Learning Outcomes

Candidates should be able to:

(a) show an understanding of and use the terms period, frequency, peak value and root-mean-square (r.m.s.) value as applied to an alternating current or voltage

(b) deduce that the mean power in a resistive load is half the maximum (peak) power for a sinusoidal alternating current

(c) represent an alternating current or an alternating voltage by an equation of the form $x = x_0 \sin\omega t$

(d) distinguish between r.m.s. and peak values and recall and solve problems using the relationship $I_{rms} = I_0 / \sqrt{2}$ for the sinusoidal case

(e) show an understanding of the principle of operation of a simple iron-core transformer and recall and solve problems using $N_s / N_p = V_s / V_p = I_p / I_s$ for an ideal transformer

(f) explain the use of a single diode for the half-wave rectification of an alternating current.

9749 (2019)

18.1 Characteristics of alternating currents

MCQs

1. A 6 V, 1.67 Hz

Peak voltage $= 2 \times 3 = 6$ V

Period $T = 6 \times 0.1 \times 10^{-3} = 6 \times 10^{-4}$

Frequency $f = \dfrac{1}{T} = 1.67$ kHz (ans)

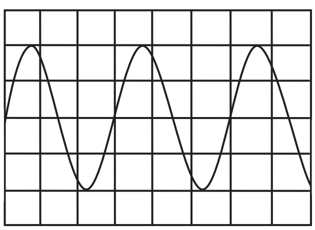
2. C $\frac{1}{2}P$

A diode would prevent the current from flowing backwards, so it would reduce the current I_{rms} by half.

$P = IV = I^2R$

So power is reduced by half if current is halved. (ans)

3. B



When the speed of rotation is doubled, frequency is doubled.

$V = V_0 \sin \varpi t$

The peak voltage will also be doubled. (ans)

4. B 12 W

From the graph, $V_{rms} = 6$V

Power dissipated $= \dfrac{V^2}{R} = 12$ W (ans)

5. B 2.65 A

$$I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\frac{0.001 \times 4^2 + 0.003 \times 2^2}{0.004}}$$

$I_{rms} = 2.65$ A (ans)

6. A 1.1 A

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{0.001 \times 4^2 + 0.003 \times 1^2}{0.004}}$$

$V_{rms} = 2.18$ V

$$I_{rms} = \frac{V_{rms}}{R} = \frac{2.18}{2} = 1.1 \text{ A}$$

$I_{rms} = 1.1$ A (ans)

7. B $\frac{1}{2}P$

$P = \langle I_0 V_0 \sin^2 \varpi t \rangle$

Mean value of $\sin^2 \varpi t$ is $\frac{1}{2}$.

$P' = \frac{1}{2} I_0 V_0 = \frac{1}{2} P$ (ans)

8. B 67.0 W

Let the peak voltage of 1 bulb be P.

$P = \frac{1}{2} {V_0}^2 R_0$

Given $P = 40$ W,

$40 = \frac{1}{2}(170)^2 R_0$

$$R_0 = \frac{80}{170^2}$$

Alternating voltage would be distributed equally between 2 bulbs connected in series. Resistance remains constant.

$$P_{total} = 2 \times \tfrac{1}{2}(V_0')^2 R_0 \quad \Rightarrow \quad P_{total} = \left(110\sqrt{2}\right)^2 \frac{80}{170^2}$$

$P = 67.0$ W (ans)

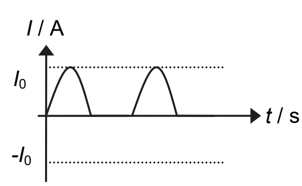
9. C 2.5 W

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{2 \times 5^2 + 3 \times 2^2}{5}} = 3.521 \text{ V}$$

$$P = \frac{V^2}{R} = \frac{3.521^2}{5} = 2.5 \text{ W} \quad \text{(ans)}$$

10. B 0.50 I^2R

Graph of I against t would be as follows:



Let I_0 be the peak current.

$P = \frac{1}{2} I_0^2 R = I^2 R$

After the diode is added,

$$P = \left(\frac{I_0}{2}\right)^2 R = \tfrac{1}{2} I^2 R \quad \text{(ans)}$$

11. **B** $\frac{1}{2}$

$$\frac{V_{a,rms}}{V_{b,rms}} = \frac{V_p\sqrt{2}}{V_p} = \frac{1}{\sqrt{2}}$$

$$P = \frac{V^2}{R}$$

Thus, P is directly proportional to the square of V.

$$\frac{P_a}{P_b} = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2} \quad \text{(ans)}$$

12. **C** 2.83 A

$P = I_{rms}^2 R$

$I_{rms}^2 5 = 20$

$I_{rms} = 2A$

$I_0 = I_{rms}\sqrt{2} = 2\sqrt{2} = 2.83 \text{ A} \quad \text{(ans)}$

13. **A** P

The power would remain changed, because $P = IV$ and V_0 is unchanged. When the frequency increases, the I_{rms} still remains as $I_{rms} = \frac{I_0}{\sqrt{2}}$. (ans)

14. **D** $\sqrt{2}I$

$P = I^2 R$

When R is halved, I^2 would double to maintain the same power.

For an alternating current, $P = I_{rms}^2 R$.

$I_{rms}^2 = 2I^2$

$I_{rms} = \sqrt{2}I \quad \text{(ans)}$

15. **C** Mean current= 0, maximum power = 2P

No current flows in the resistor, so mean current in the resistor is 0.

Power dissipated in half the maximum power.

Mean power is P. Thus, maximum power is 2P. (ans)

16. **A** 125 W

Let maximum value of the current be I_0.

$I_0 = 5$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$$

$$P = I_{rms}^2 R = \left(\frac{5}{\sqrt{2}}\right)^2 \times 10$$

$P = 125 \text{ W} \quad \text{(ans)}$

17. **A** $\frac{1}{2}P$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

$P = I_0^2 R$ and $P' = I_{rms}^2 R$

$$P' = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = \tfrac{1}{2} I_0^2 R = \tfrac{1}{2}P \quad \text{(ans)}$$

Questions – 18.1

1. (i)

The peak value of a sinusoidal alternating current is the maximum value of the current. The root-mean-square value of a sinusoidal alternating current is the value supplied by a direct current, which would supply the same power in a given resistor.
Let the r.m.s value of the current by I_{rms} . and the peak value of the current be I_0 .

Then, $I_{rms} = \frac{1}{\sqrt{2}} I_0$ (ans)

(ii)(1)
Current through the cables,
$I = \frac{P}{V} = \frac{5000}{50} = 100A$
Power lost in the cables,
$P_{lost} = I^2 R = (100)^2 (8) = 80kW$
Maximum power input of installation,

$P_{supply} - P_{lost} = 5000 - 80 = 4920 \text{ kW}$ (ans)

(ii)(2)

V_{rms} of supply $= \frac{V_0}{\sqrt{2}} = \frac{50000}{\sqrt{2}} = 35.4 \text{ kV}$
Current through cables,
$I = \frac{P}{V} = \frac{5000}{35.4} = 141 \text{ A}$
Power lost in cables,
$P_{lost} = I^2 R = (141)^2 (8) = 159 \text{ kW}$
Maximum power input of installation,

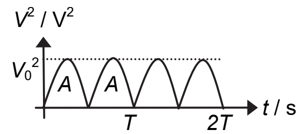
$P_{supply} - P_{lost} = 5000 - 159 = 4840 \text{ kW}$ (ans)

(iii) Power lost along the length of the transmission cable is given by $I^2 R$ where I is the current flowing in the cable and R is the resistance of the cables. Thus to minimize power loss in the transmission of electrical energy, a low current has to pass through the cable. This is achieved by transmitting the electrical energy at a high voltage, from $I = \frac{P}{V}$,
where V is the supply voltage and P is the power supply.
To attain high transmission voltages, the supply voltage must be stepped up with a transformer which is only possible by using alternating currents. The stepped up transmission voltage must then be stepped down to the required lower voltages for use by the consumers.   (ans)

2. (i)

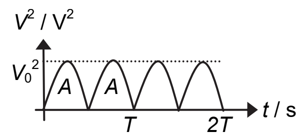
Peak value is the amplitude of the a.c. signal which root-mean-square value is equivalent to the steady direct current that converts electrical energy to other forms of energy at the same average rate as the alternating current in a given resistance (OR that which produced the same heating effect as a steady direct current of the same value.)   (ans)

(ii)



V^2 graph for Fig. 17.5

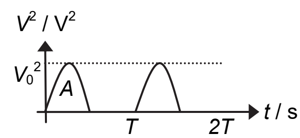
$$V_{rms} = \frac{V_0}{\sqrt{2}} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{2A}{T}}$$



V^2 graph for Fig. 17.6

Since the V^2 graph for Fig. 17.6 is the same as that for Fig. 17.5,

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{2A}{T}} = \frac{V_0}{\sqrt{2}}$$



V^2 graph for Fig. 17.7

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \sqrt{\frac{A}{T}} = \frac{1}{\sqrt{2}}\sqrt{\frac{2A}{T}} = \frac{1}{\sqrt{2}}\frac{V_0}{\sqrt{2}} = \frac{V_0}{2} \quad \text{(ans)}$$

☺ Note: Students can also prove using integration.

18.2 The transformer

MCQs

1. **B** 0.4 A

Power delivered to the primary is

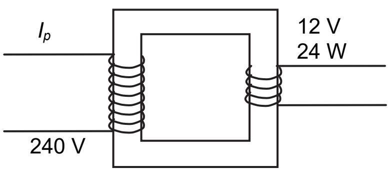
$V_p I_p = 10 \times 2 = 20\text{W}$

4 W is generated in the secondary winding.

Thus, $V_s I_s = 16$

$I_s = \frac{16}{40} = 0.4\text{A}$ (ans)

2. **B** 0.40 A



$I_{s,total} = \frac{24}{12} \times 4 = 8\text{A}$

$\frac{I_p}{I_s} = \frac{V_s}{V_p}$

$\frac{I_p}{8} = \frac{12}{240}$

$I_p = 0.40$ A (ans)

3. **B** The output power is 60 W.

$\frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\frac{V_s}{20} = \frac{1}{10}$

$V_s = 2V$

(A) is true.

For a perfect transformer,

$\frac{I_s}{I_p} = \frac{N_p}{N_s}$

$\frac{I_s}{3} = \frac{10}{1}$

$I_s = 30A$

Since efficiency is 0.9, $I_s = 30 \times 0.9 = 27$ A

(C) is true.

Output power $= 27 \times 2 = 54$ W

(B) is false.

Input power $= 3 \times 20 = 60$ W

Energy lost $= 60 - 54 = 6$ W

(D) is true. (ans)

4. **D** One alternating voltage to another alternating voltage of a different magnitude.

A transformer cannot use direct voltages as input, so (B) and (C) are false.

(A) is done by rectifiers.

(D) is true. (ans)

Questions – 18.2

1. (a)

An e.m.f. is induced in coil Y.

Faraday's law states that the induced e.m.f. is equal to the rate of change of flux linking the coil. (ans)

(b)(i) Assuming 100% efficiency, the power generated in Y equals the power delivered to X.

$V_Y I_Y = V_X I_X$

$12 \times 3 = 240 I_X$

$I_X = 0.15$ A (ans)

(b)(ii)

The bulb would not light. An e.m.f. would not be induced in coil Y because the magnetic flux in the core would not be changing. (ans)

2. (i)

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

$V_s = 10 \times 6.0 = 60$ V (ans)

(ii)

$$\langle P_{input} \rangle = \frac{P_{peak}}{2} = \frac{54}{2} = 27 \text{ W}$$

For an ideal transformer,

$\langle P_{output} \rangle = \langle P_{input} \rangle = 27$ W (ans)

(iii) $\langle P_{input} \rangle = I_{rms} V_{rms}$

$27 = I_{rms}(6.0)$

$I_{rms} = 4.5$ A (ans)

(iv)

$\langle P_{output} \rangle = I'_{rms} V'_{rms}$

$27 = I'_{rms}(60)$

$I'_{rms} = 0.45$ A (ans)

(v)

$P_{loss} = (I'_{rms})^2 R = 0.45^2(10) = 2.025$ W

Input power to secondary transformer,

$\langle P_{output} \rangle - P_{loss} = 27 - 2.025 = 24.975$ W (ans)

3. (i)

$$I = \frac{P}{V} = \frac{20 \times 10^6}{240 \times 10^3} = 83.3 \text{ A}$$

$P_{loss} = I^2 R = (83.3)^2 \times 20.0 = 1.39 \times 10^5$ W

For one day,

$E_{loss} = (1.39 \times 10^5) \times 24 \times 3600 = 1.20 \times 10^{10}$ J

$$\text{Money lost} = \frac{1.20 \times 10^{10}}{3.6 \times 10^6} \times 0.10 = \$333 \quad \text{(ans)}$$

(ii)

$$I = \frac{P}{V} = \frac{20 \times 10^6}{16 \times 10^3} = 1250 \text{ A}$$

$P_{loss} = I^2 R = (1250)^2 \times 20.0 = 3.13 \times 10^7$ W

For one day,

$E_{loss} = (3.13 \times 10^7) \times 24 \times 3600 = 2.70 \times 10^{12}$ J

$$\text{Money lost} = \frac{2.70 \times 10^{12}}{3.6 \times 10^6} \times 0.10 = \$75000$$

Money lost is much more than that in (i). (ans)

(iii)

Turns ratio = 1:15 or 15:1 (ans)

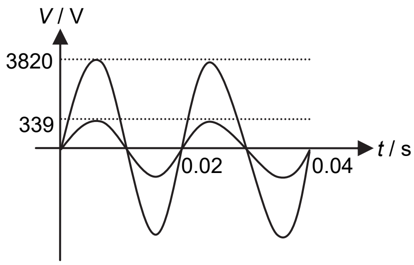
4. (a)

$$\frac{N_s}{N_p} = \frac{2700}{240} = \frac{45}{4} \quad \text{(ans)}$$

(b)

$V_p = 240\sqrt{2} = 339$ V

$V_s = 2700\sqrt{2} = 3820$ V



(ans)

(c)

$$I_2 = \frac{I_1 V_1}{V_2} = \frac{6 \times 240}{2700} = 0.533 \text{ A}$$

Hence, the power lost in the transmission line is

$$P_{lost} = I^2 R = (0.533)^2 (30) = 8.52 \text{ W}$$

Since the output of the generator is
$P = IV = 6 \times 240 = 1440$ W,

Percentage power lost

$$= \frac{8.522}{1440} \times 100\% = 0.59\% \quad \text{(ans)}$$

(d)

If the voltage were not stepped up, the current in the transmission line would be 6 A and power lost would be

$$I^2 R = 6^2 \times 30 = 1080 \text{ W}$$

Percentage power lost

$$= \frac{1080}{1440} \times 100\% = 75\% \quad \text{(ans)}$$

(e)

Cool the transmission line so that resistance is reduced.

Use thicker wires.

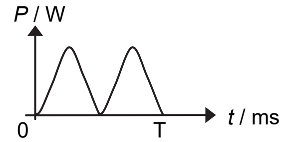
Use material of lower resistivity. (ans)

5. (i)

Faraday's law: induced electromotive force is directly proportional to the rate of change of magnetic flux linkage.

An alternating input voltage gives rise to a changing magnetic flux which links the primary coil to the secondary coil, hence inducing an e.m.f. in the secondary coil. A direct input voltage does not give rise to a changing magnetic flux and hence no e.m.f. is induced in the secondary coil. (ans)

(ii)



(ans)

(iii)

$$\text{Efficiency} = \frac{P_{output}}{P_{input}} \times 100\% \quad = \frac{V_s I_s}{V_p I_p} \times 100\%$$

$$= \frac{60 \times 4.0}{120 \times 2.5} \times 100\% = 80\% \quad \text{(ans)}$$

(iv)

Heat is lost due to the heating effect if the wires in the coils.

Heat is produced in the iron core due to eddy currents.

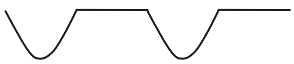
Not all flux in the iron core at the primary coil reaches the secondary coil.

Heat is produced in the iron core when energy is used in the process of magnetizing the iron core and reversing this magnetization every time the current reverses. (ans)

18.3 Rectification with a diode

MCQs

1. D



A diode prevents backward flow of current. In this case, current is directed to earth when it is positive. On the other half of the cycle, it is negative and is transmitted to the c.r.o. The graph is thus like this. (ans)

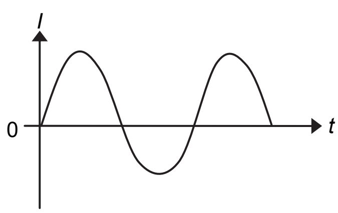
2. B 50 V

$$V_{rms} = \frac{100}{2} = 50\text{ V} \quad \text{(ans)}$$

3. D 141 V

$$V_{rms} = \frac{V_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = 141\text{ V} \quad \text{(ans)}$$

4. B



The current has difference magnitude in the positive and negative directions. When current flows in the positive direction, it passes through the branch without the 1 kΩ resistor. When it flows in the negative direction, it is forced by the diode to pass into the branch with the resistor. Thus, it has a smaller current in the negative direction. (ans)

5. B

The circuit formed by the four rectifiers is symmetrical. When the current is positive, the diodes allow it to flow through the resistor *R*. When the current is negative, the current is completely prevented from flowing through *R*. But in both cases, the currents flowing into P and Q are the same. There is no potential difference between the two points. Thus, the display in the CRO is a constant horizontal line. (ans)

Questions – 18.3

1.

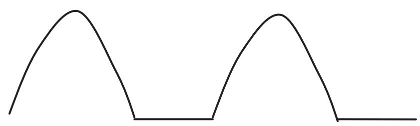
Peak value of half-wave rectified potential difference = $\sqrt{2}V_{rms} = \sqrt{2}V = 1.41V$

Vertical height of trace = $\frac{1.41}{0.50} = 2.8cm$

Period of half-wave = $\frac{1}{f} = \frac{1}{500} = 2.0ms$

Horizontal length of one cycle

= $\frac{2.0}{0.50} = 4.0\,\text{cm}$

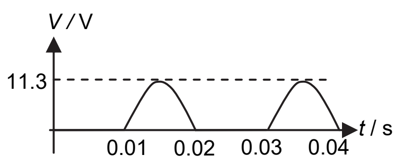


(ans)

2. (i)

The r.m.s. voltage is the equivalent value of the steady direct voltage which dissipates heat at the same average rate as the a.c. in a given resistor. (ans)

(ii)



(ans)

(iii)

The diode is a half-wave rectifier, which cuts out the availability of the negative phase to the circuit. This therefore does not permit the power of the negative phase to be available to the calculator circuit.

Voltage needed to operate the calculator is not available at all times; required voltage is only available at certain intervals of time. (ans)

Notes: