



5.1 Differentiation

Questions

[2017(Pure.V).Q1]

Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm³, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

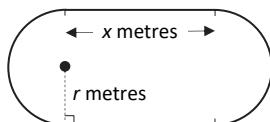
- show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$ [1]
- Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]
- Find the maximum volume of a cone that can be made by these machines. [1]

[Teachers' Comments:] In part (i), the substitution $h = (18 - r)$ led quickly to the given result in part (i), however a lot of candidates did not see this.

In part (iii), the correct differentiation and solution of $dV/dr = 0$ were frequently followed by the calculation and correct use of the second derivative to reach the given description. Those who investigated the change in sign of dV/dr around the stationary value and gave sufficient detail of their method also gained full credit.

Most of the correct values of r in part (ii) led to a correct answer to part (iii).

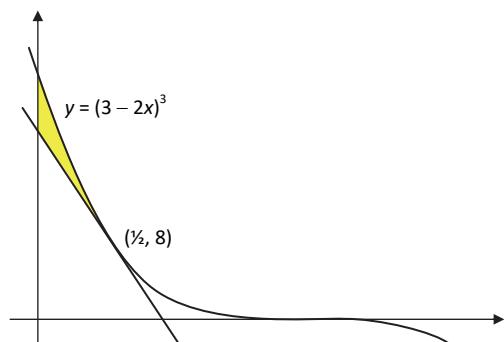
[2013(Pure.V).Q8]



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

- Show that the area, A m², of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]
- Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

[2013(Pure.V).Q10]



The diagram shows the curve $y = (3 - 2x)^3$ and the tangent to the curve at the point $(\frac{1}{2}, 8)$.

- Find the equation of this tangent, giving your answer in the form $y = mx + c$. [5]
- Find the area of the shaded region. [6]



[2013(Pure.V).Q3]

The equation of a curve is $y = \frac{1}{2}e^{2x} - 5e^x + 4x$. Find the exact x -coordinate of each of the stationary points of the curve and determine the nature of each stationary point. [6]



[2012(Pure.V).Q3]

An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]



[2012(Pure.V).Q4]

The parametric equations of a curve are

$$x = \ln(1-2t), \quad y = \frac{2}{t}, \quad \text{for } t < 0.$$

- Show that $\frac{dy}{dx} = \frac{1-2t}{t^2}$. [3]
- Find the exact coordinates of the only point on the curve at which the gradient is 3. [3]

[Teachers' Comments:] This question was probably the least well done on the paper. The basic laws of logarithms were seldom applied correctly, with the result that many candidates were unable to gain any marks. The syllabus demands of the topic of logarithms is clearly an area that needs to be concentrated upon more. For those candidates that were able to obtain the correct equation, it was pleasing to see that many gave or indicated the correct solution.

