1 Measurement

Content

Learning Outcomes

Candidates should be able to:

(a) recall the following base quantities and their units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol)

(b) express derived units as products or quotients of the base units and use the named units listed in '*Summary of Key Quantities, Symbols and Units*' as appropriate

(c) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *SI Units, Signs, Symbols and Abbreviations*, except where these have been superseded by *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*

(d) use the following prefixes and their symbols to indicate decimal sub–multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T)

(e) make reasonable estimates of physical quantities included within the syllabus

(f) show an understanding of the distinction between systematic errors (including zero errors) and random errors

(g) show an understanding of the distinction between precision and accuracy

(h) assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties (a rigorous statistical treatment is not required)

(i) distinguish between scalar and vector quantities, and give examples of each

(j) add and subtract coplanar vectors

(k) represent a vector as *two* perpendicular components.

* – Analysis, conclusions and evaluation (ACE)

measurement

Symbols, Signs & Abbreviations

Wherever symbols, signs and abbreviations are used in examination papers, the recommendation made in the ASE publication *SI Units, Signs, Symbols and Abbreviations* (1981) will be followed, except where these have been superseded by *Signs, Symbols and Systematics* (*The ASE Companion to 16-19 Science,* 2000). The units kWh, atmosphere, eV and unified atomic mass unit (u) may be used in examination papers without further explanation.

9646 (*2015*)

1.1 SI units

Understand

Physics is the *natural science* which examines basic concepts such as *energy*, *force*, and *spacetime* and all that derives from these, such as *mass*, *charge*, *matter* and its *motion*.

Since the beginning of time, people have been curious about the Universe and the natural phenomena that occur around them. They sought answers to their questions; in fact, people are still asking the same questions today.

Physics, traditionally defined the study of *matter* and *energy*, requires **experiments** to test and validate **theories**. The results of *experiments* are obtained by <u>accurate</u> measurements of its *physical quantities*.

Physicists explore the Universe. The subjects of their investigations range from stars that are millions and millions of kilometres away to particles that are smaller than atoms.

Besides discovering facts by observation and *experiment*, they must also try to discover the **laws** that summarize these facts. The *laws* discovered by *physicists* result in a better understanding of the **physical world**.

Physical quantity

<u>All</u> **physical quantities** consist of a **numerical magnitude** and a **unit** of measure.

Hence, an experimental measurement is <u>not</u> meaningful if its *physical quantity* has *magnitude*, but no associated *unit* of measure, such as:

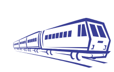
Bad Example

❶ The *length* of a table is 4 (no *unit* of measure). ✘

Worked Examples

Example 1

The length of a mass rapid transit train is 100 metres, name its physical quantity, numerical magnitude and unit of measure.



Solution

length = physical quantity
100 = numerical magnitude
metre = unit of measure **(ans)**

length = 100 m

physical quantity — numerical magnitude — unit

Example 2

[*Examined in* **2012**p1.5]

A student is given a reel of wire of diameter less than 0.2 mm and is asked to find the density of the metal.

Which pair of instruments would be most suitable for finding the **volume** of the wire?

A balance and micrometer
B metre rule and micrometer
C metre rule and vernier calipers
D micrometer and vernier calipers

Solution

$\therefore$ **B (ans)**

[Teachers' Comments:] *Many candidates misread the question and selected* **A**. *A balance and micrometer would not enable the student to calculate the volume of the wire. Candidates should ensure they read the question carefully, particularly when a critical word is given in bold.*

☺ In ancient times, **physics** was known as **natural philosophy**. The word originates from the Greek word '*physikos*'. The study of *natural philosophy* is believed to have been carried out by the Greeks some 2500 years ago.

Scientific notation

Scientific notation is a convenient way of writing large or small numbers and of performing calculations. It also quickly conveys two properties of a measurement that are useful — *significant figures* and *order of magnitude*.

Scientific notation, also known as **standard form**, is this mathematical exponential expression using powers of ten:

$$a \times 10^b$$

wherein **exponent** or **order of magnitude** b is an integer, and the **coefficient** or **absolute significant value** a is any real number $(1 \le a < 10)$, called the **significand**.

- *Scientific notation* also avoids misunderstandings due to regional differences in certain quantifiers, such as 'billion', which might indicate either 10^9 or 10^{12}.

Significant figures

It is a common mistake to confuse *scientific notation* with the idea of *significant figures*. In fact, *scientific notation* does not require using *significant figures*, nor vice versa.

Example

❶ Using *scientific notation*, in SI units the speed of light is 2.99792458×10^{8} m s^{-1} and the inch is 2.54×10^{-2} m. It would be a mistake to assume that either of these expressions indicated any uncertainty; in fact both numbers are exact.

It is possible to use *scientific notation* in conjunction with *significant figures*, but this is not mandatory and should never be assumed. It is always better to state the uncertainty explicitly. For instance, the accepted value of the unit of elementary charge can properly be expressed as $1.602176487(40) \times 10^{-19}$ coulomb, where the (40) indicates 40 counts of uncertainty in the last decimal place. If a number has been rounded off, it can be written in the form $2.5\left(\tfrac{1}{2}\right) \times 10^{-2}$ to explicitly indicate that there is a half–count of uncertainty in the last digit.

Order of magnitude

Scientific notation also enables simple *order of magnitude* comparisons. A proton's mass is 0.000 000 000 000 000 000 000 000 001 672 6 kg. If this is written as 1.6726×10^{-27} kg, it is easier to compare this mass with that of the electron, given above. The order of magnitude of the ratio of the masses can be obtained by simply comparing the exponents rather than counting all the leading zeros. In this case, '−27' is larger than '−31' and therefore the proton is four *orders of magnitude* (about 10,000 times) more massive than the electron.

Examples

❶ An electron's mass is about 0.000 000 000 000 000 000 000 000 000 000 910 938 26 kg. In *scientific notation*, this is written $9.1093826 \times 10^{-31}$ kg.

❶ The Earth's mass is about 5,973,600,000,000,000,000,000,000 kg. In *scientific notation*, this is written 5.9736×10^{24} kg.

❸ The Earth's circumference is approximately 40,000,000 m (*i.e.*, 4 followed by 7 zeroes). In *scientific notation*, this is written 4×10^{7} m. In *engineering notation*, this is written 40×10^{6} m. In SI writing style, this may be written "40 Mm" ("40 megametres").

Recall

Base Quantity

A **base quantity** is <u>chosen</u> and *arbitrarily defined* rather than being derived from a combination of other *physical quantities*.

The 6 chosen base quantities are:

Base Quantity	SI Unit	Symbol
mass	kilogram	**kg**
length	metre	**m**
time	second	**s**
current	ampere	**A**
temperature	kelvin	**K**
amount of substance	mole	**mol**

Derived quantity

A **derived quantity** is defined based on combination of *base quantities* and has a **derived unit** that is the product and/or quotient of these *base units*.

Examples

Derived quantity	Symbol	Relationship with base quantities	Relationship with base units	Derived unit
Area	A	Length × Breadth	$[\text{m}]\times[\text{m}]$	m^2
Volume	V	Length × Breadth × Height	$[\text{m}]\times[\text{m}]\times[\text{m}]$	m^3
Velocity	v	$\frac{\text{Length}}{\text{Time}}$	$\frac{[\text{m}]}{[\text{s}]}$	$\text{m}s^{-1}$
Accelera-tion	a	$\frac{\text{Velocity change}}{\text{Time}}$	$\frac{[\text{ms}^{-1}]}{[\text{s}]}$	$\text{m}s^{-2}$
Force	F	$\text{mass}\times\text{acceleration}$	$[\text{kg}]\times[\text{ms}^{-2}]$	$\text{kgm}s^{-2}$ *or* N
Work	w	$\text{force}\times\text{displacement}$	$[\text{N}]\times[\text{m}]$	Nm *or* $\text{kgm}^2\, s^{-2}$

Some *derived quantities* and *units* have special SI names.

Examples

Derived Quantity	SI Unit	Symbol	Base Units
force	newton	**N**	$kgms^{-2}$
work, energy	joule	**J**	$kgm^2 s^{-2}$
power	watt	**W**	$kgm^2 s^{-3}$
angle	radian	**rad**	–
pressure	pascal	**Pa**	$kgm^{-1} s^{-2}$
frequency	hertz	**Hz**	s^{-1}
charge	coulomb	**C**	A s
magnetic flux	weber	**Wb**	$kgm^2\ s^{-2}\ A^{-1}$
inductance	henry	**H**	$kgm^2\ s^{-2}\ A^{-2}$

Coulomb, unit of charge, is not a base unit.

☺ A *derived quantity* should be expressed only in terms of *quantities,* for example, "change in *energy* per unit *time*"; and a *derived unit* should be expressed only in terms of *units*, for example, "*joules* per *second*".

☺ The standard system of measure adopted in physics is the *french* **Système Internationale D'unités** or called **International System of Units**. It was accepted widely and abbreviated **SI** in all languages.

☺ There are altogether **7** base quantities; the last one being **luminous intensity**, with unit of **candela** [cd]; and **2** more **supplementary base quantities** – **plane angle**, with unit of **radian** [rad] and **solid angle** with unit of **steradian** [sr].

Understand

Table

A **list** or **table** of **ordered pairs** is used to represent a **relation**, particularly if the *relation* exists for sets of discrete **elements**.

When presenting **numerical data** in a **table column**, the standard way is to write the **column headings** in the form of a quotient by placing the dividend over the divisor, separating them by a horizontal line:

$$\frac{\text{name of physical quantity}}{\text{unit of measure}}$$

Example

❶ The *column heading* of speed, v is $\dfrac{\text{speed, } v}{\text{ms}^{-1}}$ or $\dfrac{v}{\text{m/s}}$.

It is conventional to have the **independent variable** to be recorded as the **leading column** in a *table*, *e.g.*, pre–selected *physical quantity*, such as length or time. The second and subsequent columns should be the **dependent variable**(s), *e.g.*, readings recorded from experiment or calculated values from previous columns.

e.g.

Pendulum length $\dfrac{l}{\text{cm}}$	Time taken for 20 oscillations $\dfrac{t}{\text{s}}$	Period $\left(T=\dfrac{t}{20}\right)$ $\dfrac{T}{\text{s}^{-1}}$
20.0	10.3	0.515
40.0	15.7	0.785
60.0	20.9	1.05

- Notice the *pendulum length readings* are recorded with the **same** number of *decimal places* (dp) according to the accuracy of the measuring instrument, *i.e.*, the metre–rule.
  - The **number of decimal places** is the number of digits given after the *decimal point*. A trailing zero counts in the number of decimal places.

Examples

❶ 1.214 has 3 dp,

❷ 120 has 0 dp, and

❸ 1.2140 has 4 dp.

- To write a number to a given number of decimal places, look at the digit in the next decimal place.
  - ✓ If it is 5 or more, increase the digit in the previous column by 1.
  - ✓ If it is 5 or less, leave the digit in the previous column.

- The *time taken* by a stopwatch however was only recorded to 1 decimal place throughout instead of the usual 2 to 3 decimal places as indicated by the time instrument. This is because the time readings recorded must take into account of the human reaction time of 0.1-0.4s in starting and stopping the stopwatch.

- The *period* of one oscillation is a calculated value taken from records of the time taken for 20 oscillations, hence, the values recorded will have the **same** number of *significant figures* (sf) as its raw score.

- The **number of significant figures** is the number of digits about which we can be certain.
  - ✓ In whole numbers, all digits before the decimal point are *significant* except final place–holding zeros.
  - ✓ In decimal fractions, all digits after the decimal point are significant except those zeros immediately following the decimal point.

Examples

❶ 0.0214 has 3 sf,

❷ 1.0 has 2 sf,

❸ 1,000 has either 1, 2, 3 or 4 sf (hence, ambiguous).

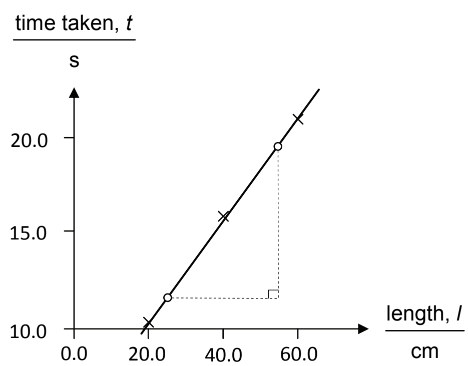
In instance like this, it would be more appropriate to use scientific notation to confirm the significance of each place value: 1.0×10^{3} has 2 sf, while 1.000×10^{3} has 4 sf.

☺ The use of **solidus notation** such as "*a*/cm" for "variable, *a* measured in units of cm" is <u>not</u> too encouraged for *column heading*.

Graph

A **graph** is an infinite set of **ordered pairs** plotted on a **Cartesian plane**, representing a relationship between the *elements* of the *ordered pair*. The *elements of the ordered pair* are usually determined by a **rule** or experiment.

- It is conventional to plot the *independent variable* on the **horizontal** (*x*–) **axis** and the *dependent variable* on the **vertical** (*y*–) **axis**.
  - To find the *x*–intercept of a graph, $y = 0$ is substituted into the *rule* and the equation (if any) solved for x.
  - To find the *y*–intercept of a graph, $x = 0$ is substituted into the *rule* and the equation (if any) solved for y.



- It is conventional that the term *graph* should refer to the *whole* diagram and that the term *curves* should refer to both curves and straight lines joining data points on the graph.

- On the *graph*, the **axes** are *directed lines*, *i.e.*, they carry an arrow, and labeled similarly to their column headings, *i.e.*, the order accuracy follows.

- The selected **scale** of the *axes* should be easy to read and large enough to enable the *curve*(*s*) to occupy at least half of the **graph grid**. The selected *scale* should usually be in multiples of 1, 2, 5 or 10 unit(s).

- Label one or more *curves* as they appear on the same *graph*.

- When determining **gradient** of the *curve* at a particular point or *straight line*, the chosen reference triangle should fall within the recorded data points and be at least half the size of the *curve*. The chosen points should not coincide with any of the recorded data points from the experiments. Label these chosen points for easy checking.

☺ By drawing a line of best fit, the effect of random errors is reduced. In addition, it increases the possibility of identifying and avoiding systematic errors, and as well as, checks for 'poor' readings.

Use

Prefixes

Prefixes are useful for expressing units of *physical quantities* that are either <u>very big</u> or <u>very small</u>.

Some of the Greek **prefixes** and their **symbols** to indicate **decimal sub–multiples** and **multiples** of the SI units are:

Prefix	Symbol	Sub–multiples & Multiples
pico	p	$10^{-12} \equiv \dfrac{1}{1\,000\,000\,000\,000}$
nano	n	$10^{-9} \equiv \dfrac{1}{1\,000\,000\,000}$
micro	μ	$10^{-6} \equiv \dfrac{1}{1\,000\,000}$
milli	m	$10^{-3} \equiv \dfrac{1}{1\,000}$
centi	c	$10^{-2} \equiv \dfrac{1}{100}$
deci	d	$10^{-1} \equiv \dfrac{1}{10}$
deca	da	$10^{1} \equiv 10$
hecto	h	$10^{2} \equiv 100$
kilo	k	$10^{3} \equiv 1000$
Mega	M	$10^{6} \equiv 1\,000\,000$
Giga	G	$10^{9} \equiv 1\,000\,000\,000$
Tera	T	$10^{12} \equiv 1\,000\,000\,000\,000$

Examples

❶ 1 μm (micrometre) $\equiv$ 10^{-3} mm (millimetre) $\equiv$ 10^{-6} m (metre)

❷ 1 kg (kilogram) $\equiv$ 10^{3} g (gram)

❸ 1,000 kJ (kilojoule) $\equiv$ 1,000,000 J (Joule) $\equiv$ 1 MJ (Megajoule)

❹ 100 cm (centimetre) $\equiv$ 1 m (metre) $\equiv$ 10 dm (decimetre)

☺ The standard for unit of **luminous intensity**, the **candela** [cd], is equal to the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} Hz and has a radiant intensity in that same direction of 1/683 watt per steradian (unit solid angle).

☺ The **candela** has replaced the standard candle or lamp as a unit of luminous intensity in calculations involving artificial lighting and is sometimes called the "new candle".

It is usual to restrict the *numerical magnitude* of the *quantity* to lie between 0.001 and 1000, by using *prefix*. Otherwise, one can also use *scientific notation*.

When *exponentials* are used in the *unit of measure*, the *exponential* applies to both the *prefix* and the *unit*.

Examples

❶ $1\ \text{cm}^2 = 1\,(\text{cm})^2 \equiv 1\,(10^{-2}\,\text{m})^2 = 10^{-4}\,\text{m}^2$

❷ $1\ \text{m}^3 = 1\,(100\,\text{cm})^3 \equiv 1\,(10^2\,\text{cm})^3 = 10^6\,\text{cm}^3$ (conversely)

It is advised that one should work with the whole SI *units*, rather than its *prefixed units*, as it is easy to be confused with its actual magnitude. For example, if unit of energy used is measured in mJ (milli-Joules), $E\,/\text{mJ}$, and unit of time taken is measured in ms (milli-seconds), $T\,/\text{ms}$, it does not automatically implied that the resultant unit of power, as measured by energy used per unit time taken, $P = \dfrac{E\,/\text{mJ}}{T\,/\text{ms}}$, to be mW (milli-Watts).

Certain *prefixes* share the same letter as some of the SI *units*. It is usual to separate the *units* using a small space, while its *prefix* is closed to its *unit*.

Examples

❶ $1\ \text{ms}^{-1} = 1$ per milli-second $= 1$ milli-hertz $\equiv 1$ mHz

❷ $1\ \text{m s}^{-1} = 1$ metre per-second

To avoid confusion, prefix is written without any space in between the prefix and unit, such as ms (milli-second), while units within a composite unit are separated by a small space, such as m s (metre second).

Estimate

Physical quantities (common)

Candidates should be able to make reasonable estimates of common physical quantities around him/her.

Examples

Physical quantity	Magnitude	Unit
Mass of an apple	100	g
Mass of a single sheet of A4 paper	5	g
Volume of an adult	0.05	m^3
Temperature of a red-hot ring on an electric cooker	800	°C
Weight of an adult	600	N
Room temperature	20	°C
Power of a hair dryer	1000	W
Energy required to bring to boil a kettleful of water	500	kJ
Resistance of a domestic filament lamp	1000	Ω
Wavelength of visible light	500	nm
Volume of an adult	0.05	m^3

Typical office paper has 80 g/m^2, therefore a typical A4 sheet (1⁄16 m^2) weighs 5 g. The unofficial unit symbol "gsm" instead of the standard "g/m^2" is also widely encountered in English speaking countries.

[*Examined in* **2013**p1.2]

This question was found to be the most difficult on the paper. Generally speaking people tend to think of 100°C as hot and 200°C is therefore very hot. In normal lighting conditions a metal will not be seen to glow until it is over 700°C so a red-hot ring is at around a temperature of 800°C. Nearly half of the candidates thought that 200°C was correct and many gave 100°C as the temperature.

[*Examined in* **2012**p1.3]

Usually the *order of magnitude* would suffice, there is no need to know the exact range of values.

Worked Examples

Example 1

Find the base units of:

(a) Velocity (b) Momentum
(c) Volume (d) Density

Solution

a. Velocity of an object is defined as the rate of change of displacement with respect to time,

i.e., velocity = displacement ÷ time

Base units of velocity = [base unit of *displacement*] / [base unit of *time*]

$= [\text{m}] / [\text{s}] = [\text{m s}^{-1}]$ **(ans)**

b. Momentum of an object is defined as the product of mass and velocity,

i.e., momentum = mass × velocity

Base units of momentum

= [base unit of *mass*] × [base unit of *velocity*]

$= [\text{kg}] \times [\text{m s}^{-1}] = [\text{kg m s}^{-1}]$ **(ans)**

c. Volume of an object is defined as the product of its length, breadth and height,

i.e., volume = length × breadth × height

Base units of volume

= [base unit of *length*]·[base unit of *breadth*]·[base unit of *height*]

$= [\text{m}]\cdot[\text{m}]\cdot[\text{m}] = [\text{m}^3]$ **(ans)**

d. Density of a regular object is defined as its mass per unit volume,

i.e., density = mass / volume

Base units of density

= [base unit of mass] / [base units of volume] $= [\text{kg}] / [\text{m}^3] = [\text{kg m}^{-3}]$
(ans)

☺ The dot "·" notation may be used in place of the multiplication "×" notation.

Example 2

Given the equation $v = \sqrt{g\lambda}$, where g is the acceleration of free fall and λ is wavelength, find the base units for v.

Solution

To determine the base units of v, we can examine its equivalent RHS:

$$\text{RHS} = \sqrt{[\text{derived unit of acceleration}] \times [\text{base unit of length}]}$$

$$= \sqrt{[\text{ms}^{-2}] \times [\text{m}]} = [\text{ms}^{-1}].$$ **(ans)**

☺ It is usual to find the base units of a derived quantity using its fundamental definition.

☺ Units, such as the joule, newton, volt and ohm, are SI units, but they are not base SI units.

☺ Unless otherwise stated, all formulae and definitions in this book are written based on standard SI units. If *pressure*, *p*, defined using *Force*/*Area* , has the SI unit of pascal (Pa), then it follows that [Pa] ≡ [N m^{-2}] or conversely, to calculate the magnitude of the *pressure* in Pa, one has to measure *Force* in newton and *Area* in square metres.

Example 3

[*Examined in* **2013**p1.1]

Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.
B The power of a domestic light is 300 W.
C The temperature of a hot oven is 300 K.
D The volume of air in a car tyre is 0.03 m^3.

ANS: **D**

[Teachers' Comment] The estimate required in the first question caused many problems. The kinetic energy of the bus might well be 106 J. Domestic lights are usually around 60 W (often less) and an oven at 300 K is not hot. The correct answer, **D**, indicates just how large a cubic metre is. Candidates could have obtained this answer by making a rough estimate of the size of a tyre and determining the volume from this.
[Publisher's Note] Practical judgement can only be acquired through thorough practices. There is no short–cut to this question-type.

1.2 Errors and uncertainties

Define

All physical readings are subjected to some **uncertainties**. These *uncertainties* give rise to **errors** from its true readings.

Error

Errors are **uncertainties** in the measured *physical quantities* that arise due to limitations of:

- the measuring instruments,
- the observer's measurement skill, and
- the experimental procedure or method used; and
- the randomly varying factors of the physical environment.

- These *errors* cause the recorded readings from the experiment to deviate from its *true value*.

- Typically, most specific *error* can be classified either as a *systematic* or *random error*.

Examples

❶ By using an instrument to measure a *physical quantity*, say current in an electrical circuit, we used an ammeter connected in series to the circuit. The ammeter will cause the circuit to have added resistance, thus contributing error to the measurement.

❷ When an observer used a rule to measure the length of a book, the position of his eye will contribute error to the measurement. This type of error is known as *parallax error*.

❸ The measurement of the acceleration due to free fall differs, if the method used is the indirect swinging pendulum, as compared to a direct falling object.

❹ Measuring the temperature of a tank of water is different in a room and as in outdoor, even though its temperature may be identical to start with. Heat is continuously being absorbed or released from the tank.

☺ **Error analysis** is the study and evaluation of **uncertainty** in measurement.

☺ In scientific studies, the word error does not carry the usual connotations of the terms *mistake* or *blunder*. *Error* in a scientific measurement means the inevitable *uncertainty* that attends all measurements. As such *errors* are not *mistakes*; one cannot eliminate them by being very careful. The best one can hope to do is to ensure that *errors* are as small as reasonable possible and to have a reliable estimate of how large they are. Most textbooks introduce additional definitions of *error*, and these are discussed later. For now, *error* is used exclusively in the sense of *uncertainty* and the two words are used interchangeably.

Systematic error

Systematic errors or *uncertainties* in measurements are usually the same each time an instrument is used, either because of errors in the instrument or through consistent incorrect use of the instrument.

Systematic errors are constant deviations of the readings in one direction from its **true value**.

Systematic errors cannot be reduced or eliminated by taking the average of repeated readings. It could be reduced by techniques such as making a mathematical correction or using calibration curves and repeat control experiments.

[*Examined in* **2013**p3.6(a)(ii)2.]

Examples

❶ zero errors of instruments;

❷ heat lost to surrounding in heat-related experiments, and

❸ background count-rate of a Gieger-Muller (G.M.) counter.

Random error

Random errors or *uncertainties* result in measurements being slightly different each time you measure the same thing or repeat an experiment to check on previous results.

Random errors refer to the scatter of readings about a *mean value*, usually the sum of the *true value* and all its *systematic errors*.

Random errors are of varying sign and magnitude and cannot be eliminated, but averaging repeated readings is the best way to minimize these *errors*.

[*Examined in* **2013**p3.6(a)(ii)1.]

Examples

❶ human reaction time;

❷ parallax error; and

❸ imperfect materials used.

☺ *Random error* is always present in a measurement. It is caused by inherently unpredictable fluctuations in the readings of a measurement apparatus or in the experimenter's interpretation of the instrumental reading. *Random errors* show up as different results for ostensibly the same repeated measurement. They can be estimated by comparing multiple measurements, and reduced by averaging multiple measurements.

[*Examined in* **2013**p3.6(a)(iii)]

Distinguish

ALL *errors* can be minimized by specific methods but can never be totally eliminated. One can further qualify the value of collected data by:

Precision

Precision is the degree of agreement among a series of *measurements* of the same *physical quantity*.

- It is a measure of the *reproducibility* of results rather than their correctness. Good *precision* means the readings are mostly very close to their mean, and is associated with *small random errors.*

Accuracy

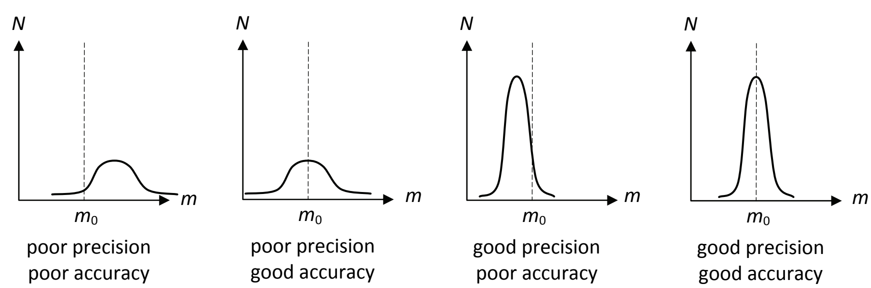
Accuracy is the degree of agreement between the experimental result and its *true value*.

- Good *accuracy* means the reading or the mean of a set of readings is very close to the true value, and is associated with *small systematic errors.*

Examples

❶ A physical quantity m is measured repeatedly. For each recorded m, its frequency of occurrence, N is plotted. The true value of the quantity is m_0.

To deduce the *quality* of collected data in terms of its *precision* and *accuracy*, possible results are as shown:

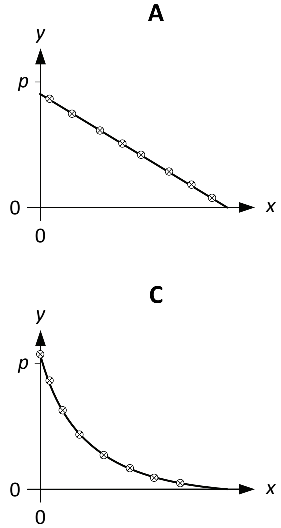


❷ If the measurement is close to the true value, it is considered **accurate**, otherwise **inaccurate**.

If repeated measurements are close to each other, it is considered **precise**, otherwise **imprecise**.

[*Examined in* **2013**p1.1, **2012**p1.1]

❸ Variables x and y are related by the equation $y = p - qx$ where p and q are constants. Values of x and y are measured experimentally. The results contain a systematic error. Which graph best represents these results?



∴ The answer is **A**

[Teachers' Comments:] ***A*** *is correct as it shows no random error but a systematic error with the line not going through p.* ***B*** *was popular but it shows random error without systematic error.*

[*Examined in* **2012**p1.6]

Assess (mathematically)

Error

The **least unit of measurement** is the smallest division available on the measuring equipment. The **error** or **uncertainty** is the difference between the *measured value* x and the *true value* and is half the *least unit of measurement* (LUM) = Δx, *i.e.*, the measured quantity and its associated **absolute error** is therefore written as " $x \pm \Delta x$ " (the *absolute error* carries only 1 or 2 sf (significant figures)).

Example

❶ Given measured gravitational force constant, $g = 9.824 \pm 0.02385$ ms^{-2}, the presentation should be

$g = 9.82 \pm 0.03$ ms^{-2} (rounded up to 1 significant figure)

☺ ***Common mistake*** in estimating errors is by rounding down the error to one significant figure and presented as such. This will reduce the overall confidence as to whether the final error or uncertainty quoted is all encompassing of its componential uncertainties.

- When this measured *physical quantity* is manipulated mathematically with/out other measured *physical quantities*, a new derived error has to be assigned based on the following rules:

Absolute, fractional and percentage errors

- The **absolute**, **fractional** and **percentage errors** of a quantity $x \pm \Delta x$ are given by:

Δx , $\dfrac{\Delta x}{x}$ and $\dfrac{\Delta x}{x} \times$ 100% respectively.

Example

❶ Given a length of 1.2 km, find its *absolute*, *fractional* and *percentage errors.*

Length $= 1.2$ km
LUM $= 0.1$ km

Absolute error $= 0.05$ km,

Fractional error $= \dfrac{0.05}{1.2} = 0.04$, and

Percentage error $= \dfrac{0.05}{1.2} \times 100\% = 4\%$. **(ans)**

Sum and difference

- When 2 or more quantities are added or subtracted from one another, its associated *error* to the **sum or difference** is the sum of the *absolute errors* of those quantities:

$$r = x \pm y \quad \Rightarrow \quad \Delta r = \Delta x + \Delta y$$

Example

❶ Given $x = 58.5 \pm 0.5$ mm and $y = 17.0 \pm 0.5$ mm, find $r = x \pm y$.

$r = x \pm y \;\Rightarrow\; \Delta r = \Delta x + \Delta y \;=\; 0.5 + 0.5 \;=\; 1.0 \;=\; 1$ mm (1sf)

$\therefore$ the sum, $r = x + y = (58.5 + 17.0) \pm 1 = 75.5 \pm 1 = 76 \pm 1$ mm

and, the difference, $r = x - y = (58.5 - 17.0) \pm 1 = 41.5 \pm 1$

$= 42 \pm 1$ mm **(ans)**

☺ Note the *decimal place* of the final value should follow that of its *absolute error*.

Product and quotient

When 2 or more quantities are multiplied or divided, its associated *error* to the **product or quotient** is the sum of the *fractional or percentage errors* of those quantities:

$$r = xy \quad \text{or} \quad r = \frac{x}{y} \quad \Rightarrow \quad \frac{\Delta r}{r} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

$$\text{or} \quad \frac{\Delta r}{r} \times 100\% = \frac{\Delta x}{x} \times 100\% + \frac{\Delta y}{y} \times 100\%$$

Example

❶ Given two quantities, $x = 58.5 \pm 0.5$ mm and $y = 17.0 \pm 0.5$ mm, find $r = xy$ and $r = \frac{x}{y}$.

Working in fractional errors:

The product, $r = xy \;\Rightarrow\; r = 58.5 \times 17.0 = 994.5$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta x}{x} + \frac{\Delta y}{y} \;\Rightarrow\; \frac{\Delta r}{994.5} = \frac{0.5}{58.5} + \frac{0.5}{17.0}$$

= 0.00858 + 0.0294 = 0.0380

$\Rightarrow \Delta r = 0.0380 \times 994.5 = 37.8 = 40$ **(1sf, rounding up)**

$\therefore\; r = 990 \pm 40$ mm **(ans)**

The quotient, $r = \frac{x}{y} \;\Rightarrow\; r = \frac{58.5}{17.0} = 3.44$

$$\Rightarrow \frac{\Delta r}{r} = \frac{\Delta x}{x} + \frac{\Delta y}{y} \;\Rightarrow\; \frac{\Delta r}{3.44} = \frac{0.5}{58.5} + \frac{0.5}{17.0}$$

= 0.00858 + 0.0294 = 0.0380

$\Rightarrow \Delta r = 0.0380 \times 3.44 = 0.131 = 0.2$ **(1sf, rounding up)**

$\therefore\; r = 3.4 \pm 0.2$ mm **(ans)**

Or working in percentage errors:

The product, $r = xy \;\Rightarrow\; r = 58.5 \times 17.0 = 994.5$

$$\Rightarrow \frac{\Delta r}{r} \times 100\% = \frac{\Delta x}{x} \times 100\% + \frac{\Delta y}{y} \times 100\%$$

$$\Rightarrow \frac{\Delta r}{994.5} \times 100\% = \frac{0.5}{58.5} \times 100\% + \frac{0.5}{17.0} \times 100\%$$

= 0.858% + 2.94% = 3.80%

$\Rightarrow \Delta r = 3.80\% \times 994.5 = 37.8 = 40$ **(1sf, rounding up)**

$\therefore\; r = 990 \pm 40$ mm **(ans)**

The quotient, $r = \frac{x}{y} \;\Rightarrow\; r = \frac{58.5}{17.0} = 3.44$

$$\Rightarrow \frac{\Delta r}{r}\times 100\% = \frac{\Delta x}{x}\times 100\% + \frac{\Delta y}{y}\times 100\%$$

$$\Rightarrow \frac{\Delta r}{3.44}\times 100\% = \frac{0.5}{58.5}\times 100\% + \frac{0.5}{17.0}\times 100\%$$

$$= 0.858\% + 2.94\% = 3.80\%$$

$\Rightarrow \Delta r = 3.80\% \times 3.44 = 0.131 = 0.2$ **(1sf, rounding up)**

$\therefore\ r = 3.4 \pm 0.2$ mm **(ans)**

☺ Note working with *percentage errors* is safer in the long run, as it avoids nasty confusion with their *absolute errors*.

Exponential

The *fractional or percentage uncertainty* of an **exponential** is the *product* of the *exponent* and the *fractional uncertainty* of the base, *i.e.,*

$$r = x^n \Rightarrow \frac{\Delta r}{r} = n\frac{\Delta x}{x}$$

$$\text{or } \frac{\Delta r}{r}\times 100\% = n\frac{\Delta x}{x}\times 100\%$$

Example

❶ Given $x = 15.5 \pm 0.5$ mm, find $r = \sqrt{x}$.

Working in fractional errors:

$$r = \sqrt{x} \Rightarrow r = x^{\frac{1}{2}} = (15.5)^{\frac{1}{2}} = 3.94$$

$$\Rightarrow \frac{\Delta r}{r} = n\frac{\Delta x}{x} \Rightarrow \frac{\Delta r}{3.94} = \tfrac{1}{2}\times\frac{0.5}{15.5} = 0.0161$$

$\Rightarrow \Delta r = 0.0161 \times 3.94 = 0.0635 = 0.07$ **(1sf, rounding up)**

$\therefore\ r = 3.94 \pm 0.07$ mm **(ans)**

Or working in percentage errors:

$$r = \sqrt{x} \Rightarrow r = x^{\frac{1}{2}} = (15.5)^{\frac{1}{2}} = 3.94$$

$$\Rightarrow \frac{\Delta r}{r}\times 100\% = n\frac{\Delta x}{x}\times 100\%$$

$$\Rightarrow \frac{\Delta r}{3.94}\times 100\% = \tfrac{1}{2}\times\frac{0.5}{15.5}\times 100\% = 1.61\%$$

$\Rightarrow \Delta r = 1.61\% \times 3.94 = 0.0635 = 0.07$ **(1sf, rounding up)**

$\therefore\ r = 3.94 \pm 0.07$ mm **(ans)**

[*Examined in* **2012**p1.2]

Worked Examples

Example 1

In an experiment to determine the power, *P*, dissipated in a resistor of a circuit, the potential difference of the resistor *V* and the resistance *R* are measured. The uncertainty in the measurement of *V* was estimated to be 4%, and that of *R*, 3%. What would be the percentage uncertainty in the value of *P* obtained?

Solution

$$P = \frac{V^2}{R} \quad \Rightarrow \quad \frac{\Delta P}{P} = 2\frac{\Delta V}{V} + \frac{\Delta R}{R}$$

$\therefore$ Percentage uncertainty of P $= 2(4) + 3 = 11\%$ **(ans)**

Example 2

[*Examined in* **2013**p1.5]

A micrometer screw gauge is used to measure the diameter of a small uniform steel sphere. The micrometer reading is 5.00 mm ± 0.01 mm. What will be the percentage uncertainty in a calculation of the volume of the sphere, using these values?

A 0.2% **B** 0.4% **C** 0.6% **D** 1.2%

ANS: **C**

[Teachers' Comment] Many candidates thought that the problem was just one of radius and diameter, but that is missing the point. If there is a 0.2 % uncertainty in the diameter, then there will be a 0.6 % uncertainty in the volume $\left[V \propto r^3\right]$.

1.3 Scalars and vectors

Distinguish

Scalar quantity

Scalar quantities are *physical quantities* in which the *magnitude* is stated, but the *direction* is either not applicable or specified.

- *Scalars* may be added together by simple arithmetic.

Examples

[*Examined in* **2012**p1.4]

1. Length, charge, volume, mass, time, distance travelled, density, speed, work done, energy, kinetic energy, potential energy, pressure, power, temperature, heat capacity, latent heat, frequency, wavelength, potential difference, and etc.

Vector quantity

Vector quantities are *physical quantities* in which both the *magnitude* and the *direction* must be stated.

- When *vectors* are added together, both their *magnitude* and *direction* must be taken into account.

Examples

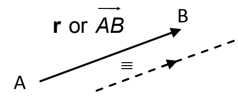
[*Examined in* **2012**p1.4]

1. Weight, **area**, force, moment of force, torque, gravitational field, electric field, magnetic field, displacement, velocity, acceleration, angular velocity, momentum, and etc.

Vectors

Notation

- On diagram, a **vector** can be <u>represented</u> by a line with an arrow-head, where the scaled length of the line represents the *magnitude* of the vector and the arrow-head shows the *direction*.



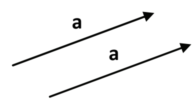
  - Although a *vector* has both *magnitude* and *direction*, it <u>need not</u> have a fixed *position*.
  - A *vector* is <u>not</u> altered if it is displaced parallel to itself, as long as its *magnitude* is <u>not</u> changed.

- A **vector symbol** is used to <u>distinguish</u> between *vector* and *scalar quantities* in formulae and on diagrams:
  - bold-face letters, such as **a** or **r** in printed works only;
  - light-face underlined letters, such as $\underline{a}$ or $\underline{r}$ or light-face letters capped by a half-arrow, such as $\vec{a}$ or $\vec{r}$ in hand-written texts; or
  - its <u>initial</u> and <u>final</u> positions, capped with an arrow pointing from *initial* towards its *final* position, such as $\overrightarrow{AB}$ or $\overrightarrow{PQ}$.

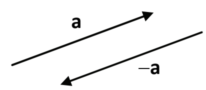
- **Scalars** are represented only by light-face letters, such as *a* or *r*.

- The **magnitude** of a *vector*, denoted by $|\vec{a}|$ (modulus of $\vec{a}$), is itself a scalar, *i.e.*, $|\vec{a}| = a$.

- **Equal vectors** are *vectors* that have the <u>same</u> *magnitude* and *direction*, but <u>need not</u> be from the same *position*.



- A *vector* that has the <u>same</u> *magnitude* but <u>opposite</u> *direction* as another *vector*, say **a** , is known as the **negative vector** of **a** , denoted by $-\mathbf{a}$.



- A *vector* that has <u>zero</u> *magnitude* and <u>no</u> *direction* is known as a **zero vector**.
  - A *zero vector* is denoted by **0**.

- A *vector* that has a *magnitude* of <u>unity</u> is also known as a **unit vector**.
  - The *unit vector* that has the same *direction* as another non-zero *vector*, say, $\mathbf{p}$, is denoted by $\hat{\mathbf{p}}$, where $\hat{\mathbf{p}} = \dfrac{\mathbf{p}}{|\mathbf{p}|}$.

- **Coplanar** *vectors* are *vectors* that lie in the same plane.

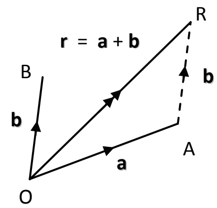
☺ Many candidates did not recognise <u>displacement</u> as a vector or thought that kinetic energy was a vector. [*Examined in* **2012**p1.4]

Vector addition

- If a *vector*, such as $\overrightarrow{OA}$ or **a**, is <u>added</u> to another *vector*, such as $\overrightarrow{OB}$ or **b**, the result is another *vector*, such as $\overrightarrow{OR}$ or **r**, written as $\overrightarrow{OA}$ + $\overrightarrow{OB}$ $\overrightarrow{OB}$ = **a** + **b** = $\overrightarrow{OR}$ = **r**.

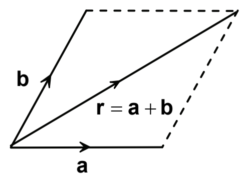
  ❶ **Triangle law**

  The sum of $\mathbf{a}$ and $\mathbf{b}$ is obtained by placing the starting point of $\mathbf{b}$ on the ending point of $\mathbf{a}$ and then joining the starting point of $\mathbf{a}$ <u>to</u> the ending point of **b**.



  ❷ **Parallelogram law**

  The sum of $\mathbf{a}$ and $\mathbf{b}$ is the <u>diagonal</u> (denoted by $\mathbf{r}$) of the parallelogram for which $\mathbf{a}$ and $\mathbf{b}$ are adjacent sides (outwardly directed).

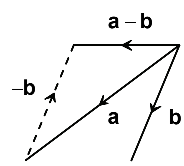


- The **resultant vector** or the <u>sum</u> of vectors, **r**, is then the *vector* that starts where **a** begins and ends where **b** ends.

  $\Rightarrow$ $\mathbf{r} = \mathbf{a} + \mathbf{b} \equiv \mathbf{b} + \mathbf{a}$

Vector subtraction

- When one vector is <u>subtracted</u> from another, it is to find the <u>difference</u> of two *vectors*, say, $\mathbf{a}$ and $\mathbf{b}$, and is denoted by $\mathbf{a}-\mathbf{b}$. It is <u>equivalent</u> to *vector addition*, with the orientation of the second *vector* reversed. Thus, the <u>difference</u> of $\mathbf{a}$ and $\mathbf{b}$ can also be written as

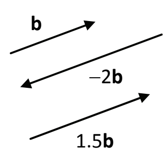


  $$\mathbf{a}-\mathbf{b} \equiv \mathbf{a}+(-\mathbf{b}).$$

Scalar multiplication

- When a *vector* **b**, is <u>multiplied</u> by a number (scalar, say *k*), = *k***b**, it is the repeated addition of the <u>same</u> *vector*, so the *vector* <u>changes</u> its *magnitude* only.

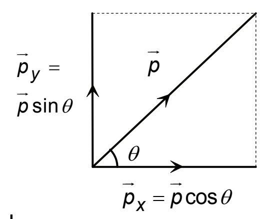
  - If *k* is <u>negative</u>, its *direction* changes too.



☺ **Scalar multiplication** is <u>different</u> from *dot* and *cross products* of vectors.

Vector resolution

- The process of breaking up a *vector*, say, $\vec{p}$, into two perpendicular (orthogonal) components, usually along the *x*-axis and *y*-axis, is known as the **resolution of vector**.



- The two perpendicular components are

  $\mathbf{p}_y = |\vec{p}|\sin\theta$ and $\mathbf{p}_x = |\vec{p}|\cos\theta$, where θ is the angle between $\vec{p}$ and the $\vec{p}_x$.

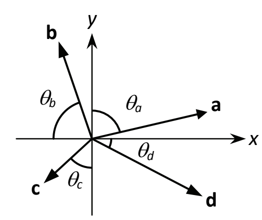
- Sometimes, it is more convenient to resolve the *vector* along and perpendicular to say, an inclined plane.

- $\mathbf{p} = \mathbf{p}_x + \mathbf{p}_y$

- $p = \sqrt{p_x^{\,2} + p_y^{\,2}}$, in the direction of $\tan^{-1}\left(\dfrac{p_y}{p_x}\right)$.

Example

❶ 4 coplanar vectors **a**, **b**, **c** and **d** are given. To find the resultant vector **R**, each vector is resolved into its orthogonal components in the vertical (*y*) and horizontal (*x*) directions.



Thus,

$$\mathbf{R}_x = +\mathbf{a}\sin\theta_a - \mathbf{b}\cos\theta_b - \mathbf{c}\sin\theta_c + \mathbf{d}\cos\theta_d$$

$$\mathbf{R}_y = +\mathbf{a}\cos\theta_a + \mathbf{b}\sin\theta_b - \mathbf{c}\cos\theta_c - \mathbf{d}\sin\theta_d$$

The *resultant vector* is $R = \sqrt{R_x^{\,2} + R_y^{\,2}}$, in the direction of $\tan^{-1}\left(\dfrac{R_y}{R_x}\right)$.

Laws of vector operation

- If $\mathbf{a}$, b and c are *vectors* and *m* and *n* are *scalars*, then,

❶ $\mathrm{a} + \mathbf{b} = \mathrm{b} + \mathbf{a}$

❷ $\mathrm{a} + (\mathbf{b} + \mathbf{c}) = (\mathrm{a} + \mathbf{b}) + \mathrm{c}$

❸ $\mathrm{a} + (-\mathbf{a}) = 0 = (-\mathbf{a}) + \mathrm{a}$

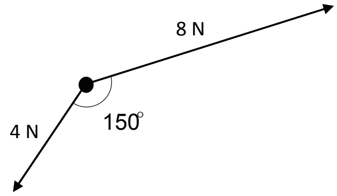
❹ $(mn)(\mathbf{a}) = (m)(n\mathrm{a})$

❺ $(m+n)(\mathbf{a}) = m\mathrm{a} + n\mathrm{a}$

Worked Examples

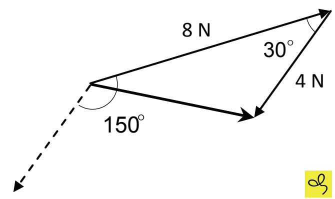
Example 1

Forces of 4 N and 8 N act at a point as shown in the diagram. Calculate the magnitude of the resultant force.



Solution

Let the magnitude of the resultant force be *x*.



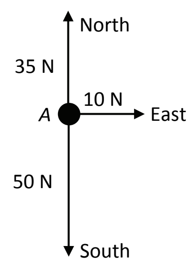
Using Cosine rule,

$$x^2 = 4^2 + 8^2 - 2(4)(8)\cdot\cos 30°$$

$x = 5.0$ N **(ans)**

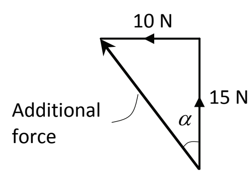
Example 2

Three coplanar forces with magnitude 10 N, 35 N and 50 N act on a body at A in the directions as shown in the diagram. All bearings are taken to be the clockwise angle from the Northward direction. Find the magnitude and bearing of the additional force that is required to maintain body A in equilibrium.



Solution

For the forces to be in equilibrium, the additional force when resolved, should have a horizontal component of 10 N acting westward and a vertical component of (50 – 35) N = 15 N acting northward.



Magnitude of the additional force $= \sqrt{10^2 + 15^2} = 18.0\text{N}$ (3 sf)

$$\tan\alpha = \frac{10}{15} \Rightarrow \alpha = 33.69°$$

Bearing of the additional force = $360° - 33.69° = 326.3°$ (1 dp)

$\therefore$ The magnitude of the additional force required is 18.0 N, with bearing of $326.3°$. **(ans)**

Worked Problems

Example 1

(a) The length, mass and temperature are examples of base quantities. State three other base quantities.

(b) Explain why velocity is said to be a derived quantity.

(c) An electron of mass m has kinetic energy E. The de Broglie wavelength λ of the electron may be represented by the expression,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

(i) Determine the base units of the kinetic energy E of the electron.

(ii) Use the above expression to show that the unit of h is $\text{kgm}^2\,\text{s}^{-1}$.

Solution

(a) The other three base quantities are time (s), electric current (A) and amount of substance (mol). **(ans)**

(b) *Velocity*, defined as the change in displacement per unit time, is obtained from the quotient of two base quantities, displacement (length) and time. Hence, it is said to be a derived quantity. **(ans)**

(c) (i) $E = \frac{1}{2}mv^2$

units of E = (units of m) × (units of v)^2

$= \text{kgm}^2\,\text{s}^{-2}$ **(ans)**

(ii) $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow h = \lambda\sqrt{2mE}$

Unit of h $= \text{m}\sqrt{\text{kg}\left(\text{kgm}^2\,\text{s}^{-2}\right)} = \text{m}\left(\text{kgms}^{-1}\right) = \text{kgm}^2\,\text{s}^{-1}$ **(ans)**

Example 2

A student uses a vernier caliper to measure the external diameter of a glass container. However, when taking the measurements, he makes the error of applying different pressures when closing the gap of the vernier calipers.

(a) State and explain whether this mistake introduces a systematic error or random error into the readings of the diameter of the container.

(b) Explain why the readings may be accurate but not precise.

(c) Suggest a way to eliminate or reduce the error.

Solution

(a) It causes a random error in the readings. As different pressures are applied in closing the gap of the vernier calipers, the readings will be inconsistent and fluctuate around the actual reading. **(ans)**

(b) The readings may be accurate as they will scatter around the actual reading but over a considerably large range, hence not precise. **(ans)**

(c) The error can be reduced by taking the average of repeated readings of the diameter of the container. **(ans)**

1.4 General Teachers' Comments

2013 Teachers' Comments

MCQs: The test showed a wide variation in the standard of the candidates. 3–5 questions were found to be particularly easy, while 3–5 questions were found to be difficult. It is challenging for candidates to manage their time when completing multiple choice papers. Not all the questions take the same time to analyse, so candidates should use time gained on the simpler questions to give further thought to more complex questions. Candidates should be advised never to take a disproportionate length of time over one question, but if a question is omitted at any stage it is crucial that a corresponding space is left on the answer sheet.

2012 Teachers' Comments

MCQs: With so many questions to be answered within the time limit, accurate and quick working is essential. Candidates must know that they should not spend too long on any one question. Many questions need written working if candidates are to maintain accuracy, and space is provided on the paper for this. Candidates found this paper difficult. Several questions had a proportion of correct answers in the range 20–30% and this may suggest that the candidates were guessing.

The candidates found the subject area of electricity to be particularly difficult, and would benefit from further practice solving questions on electricity. Candidates should also be encouraged to take particular care when reading values from graphs; it is important to look at the label of the axis as well as the numbers to ensure that power–of–ten or unit errors are not made.

Structured Questions: There was no real evidence from any of the candidates of a shortage of time. There were many scripts where the total mark was very low. Candidates should be encouraged to develop a thorough knowledge of the basic requirements outlined in the syllabus. This knowledge is a foundation for the candidates to gain credit. There were large sections of many of the questions where the vast majority of the candidates made little or no attempt. There were many examples where the candidate did not give an answer that was linked to the key command words (*e.g.*, 'state', 'explain') in the question. The answer given then failed to answer the question.