Long Questions

Learning objectives implemented in the period:

Start of 2001–2005

1. [Speed] [2001–2004]

Method

Approach I – data analysis

There are 3 towns in a city. Rahman wants to drive from town A to town C. After travelling 1/3 of the journey at an average speed of 72 km/h for 45 minutes, he reaches town B to have his lunch. He intends to take 1/2 hour for lunch. Then he will carry on with the rest of the journey at an average speed of 90 km/h. he intends to reach town C at 1.15 p.m.

(a) $\therefore$ Let d_{BC} be the distance between town B and town C.

Let d_{AB} be the distance between town A and town B.

$d_{AB} = \text{speed}_{AB} \times \text{time taken}_{AB}$

$= 72 \text{ km/h} \times 45 \text{ mins}$

$= 72 \text{ km/h} \times \frac{45}{60} \text{ hr}$

$= 72 \times \frac{45}{60} \text{ km}$

$= \cancel{72}\,\cancel{36}\,18 \times \frac{\cancel{45}\,\cancel{15}\,3}{\cancel{60}\,\cancel{20}\,\cancel{4}\,\cancel{2}\,1}$

$= 18 \times 3$

$= 54 \text{ km}$

The total distance from town A to town C

$= 54 \times 3$

$= 162 \text{ km } (d_{AB} + d_{BC})$

$\therefore$ **The distance between town B and town C is**

$d_{BC} = (d_{AB} + d_{BC}) - d_{AB}$

$= 162 - 54$

$= 108 \text{ km}$ **(ans)**

(b) $\therefore$ Let H be the time when Rahman is to leave town A.

He intends to continue the rest of the journey at 90 km/h,

The time he needs is

$\text{Time taken}_{BC} = \frac{\text{distance travelled}}{\text{average speed}}$

$= \frac{108 \text{ km}}{90 \text{ km/h}}$

$= \frac{90+18}{90}$

$= 1 + \frac{18}{90}$

$= 1 + \frac{\cancel{18}\,\cancel{2}\,1}{\cancel{90}\,\cancel{10}\,5}$

$= 1 + \frac{1}{5}$

$= 1 \text{ hr} + \frac{1}{5} \times 60 \text{ min}$

$= 1 \text{ hr} + \frac{1}{\cancel{5}\,1} \times \cancel{60}\,12 \text{ min}$

$= 1 \text{ hr } 12 \text{ min}$

The total time taken for the whole journey plus rest is

$= 45 \text{ min} + ½ \text{ hour} + 1 \text{ hr } 12 \text{ min}$

$= 45 \text{ min} + 30 \text{ min} + 1 \text{ hr} + 12 \text{ min}$

$= 1 \text{ hr} + 45 \text{ min} + 30 \text{ min} + 12 \text{ min}$

$= 1 \text{ hr} + 45 \text{ min} + 30 \text{ min} + 12 \text{ min}$

$= 1 \text{ hr} + 87 \text{ min}$

$= 1 \text{ hr} + 60 \text{ min} + 27 \text{ min}$

$= 1 \text{ hr} + 1 \text{ hr} + 27 \text{ min}$

$= 2 \text{ hr} + 27 \text{ min}$

∴ The time for Rahman to leave town A is

$H = 1.15$ p.m. – 2h 27m

$= 12.15$ p.m. – 1h 27m

$= 11.15$ a.m. – 27m

$= 11.00$ a.m. – 12m

$= 11.00$ a.m. – 12m

$= 10.48$ a.m. **(ans)**

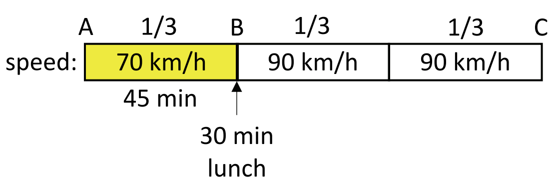
Approach II – model

There are 3 towns in a city. Rahman wants to drive from town A to town C. After travelling 1/3 of the journey at an average speed of 72 km/h for 45 minutes, he reaches town B to have his lunch. He intends to take 1/2 hour for lunch. Then he will carry on with the rest of the journey at an average speed of 90 km/h. he intends to reach town C at 1.15 p.m.

(a) ∴ Let d_{BC} be the distance between town B and town C.

Let d_{AB} be the distance between town A and town B.

The information on the speed, distance and time is as shown below.



Deduce that (given speed and time, can find distance),

$d_{AB} = \text{speed}_{AB} \times \text{time taken}_{AB}$

$= 72 \text{ km/h} \times 45 \text{ mins}$

$= 72 \text{ km/h} \times \dfrac{45}{60} \text{ hr}$

$= 72 \times \dfrac{45}{60} \text{ km}$

$= \cancel{72}\ \cancel{36}\ 18 \times \dfrac{\cancel{45}\ \cancel{15}\ 3}{\cancel{60}\ \cancel{20}\ \cancel{4}\ \cancel{2}\ 1}$

$= 18 \times 3$

$= 54$ km

The total distance from town A to town C

$= 54 \times 3$

$= 162 \text{ km } (d_{AB} + d_{BC})$

∴ The distance between town B and town C is

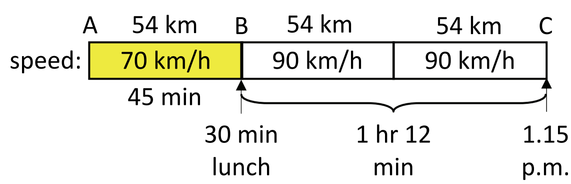
$d_{BC} = (d_{AB} + d_{BC}) - d_{AB}$

$= 162 - 54$

$= 108$ km **(ans)**

(b) ∴ Let H be the time when Rahman is to leave town A.

The information on the speed, distance and time is as shown below.



He intends to continue the rest of the journey at 90 km/h,

The time he needs is

$\text{Time taken}_{BC} = \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{108 \text{ km}}{90 \text{ km/h}}$

$= \dfrac{90 + 18}{90}$

$= 1 + \dfrac{18}{90}$

$= 1 + \dfrac{\cancel{18}\ \cancel{2}\ 1}{\cancel{90}\ \cancel{10}\ 5}$

$= 1 + \dfrac{1}{5}$

$= 1 \text{ hr} + \dfrac{1}{5} \times 60 \text{ min}$

$= 1 \text{ hr} + \dfrac{1}{\cancel{5}\,1} \times \cancel{60}\,12$ min

$= 1$ hr 12 min

The total time taken for the whole journey plus rest is

$= 45$ min + ½ hour + 1 hr 12 min

$= 45$ min + 30 min + 1 hr + 12 min

$= 1$ hr + 45 min + 30 min + 12 min

$= 1$ hr + 45 min + 30 min + 12 min

$= 1$ hr + 87 min

$= 1$ hr + 60 min + 27 min

$= 1$ hr + 1 hr + 27 min

$= 2$ hr + 27 min

∴ The time for Rahman to leave town A is

H = 1.15 p.m. – 2h 27m

= 12.15 p.m. – 1h 27m

= 11.15 a.m. – 27m

= 11.00 a.m. – 12m

= 11.00 a.m. – 12m

= 10.48 a.m. **(ans)**

☺ **CheckBack**

(a) If the answer is 108 km,

Distance d_{AB} = half of d_{BC}

$= \dfrac{108}{2} = 54$ km

Time taken $= \dfrac{\text{distance travelled}}{\text{average speed}}$

$= \dfrac{54 \text{ km}}{72 \text{ km/h}}$

$= \dfrac{\cancel{54}\ \cancel{27}\ \cancel{9}\ 3}{\cancel{72}\ \cancel{36}\ \cancel{12}\ 4}$

= ¾ hr

= 45 min **(checked)**

(b) If the answer is 10.48 a.m., adding the times for the first 45 mins journey and 30 mins for lunch,

10.48 a.m. + 45 min + 30 min

= 11.33 a.m. + 30 min

= 12.03 p.m.

From 12.03 p.m. to 1.15 p.m., the time elapsed is 1 hr 12 min.

Rahman took 1hr 12 min (or 1.2hr) to complete the last part of the journey,

$\text{Speed}_{BC} = \dfrac{\text{distance travelled}}{\text{time elapsed}}$

$= \dfrac{108 \text{ km}}{1.2 \text{ hr}} = \dfrac{108 \times 10}{1.2 \times 10} = \dfrac{1080}{12}$

$= \dfrac{\cancel{1080}\ \cancel{540}\ \cancel{270}\ 90}{\cancel{12}\ \cancel{6}\ \cancel{3}\ 1}$

= 90 km/h (by long division)

(checked)

☺ **Exam Report**

This question presented little problem to many candidates. For those candidates that did not give the correct answer, it was frequently due to problem involved in interpreting the question itself.

2. [Speed] [2001–2004]

Method

Approach I – data analysis

In one school's walkathon, two competitors Raju and Gopal started at 7.30 a.m. Raju's average speed was 20 m/min faster than Gopal. By the time Raju completes the walkathon in 30 minutes, Gopal has only walked 5/6 of the distance.

(a) ∴ Let *H* be the time when Gopal completes the walkathon.

In 30 minutes, Gopal has only completed 5/6 of the distance.

$\Rightarrow \frac{5}{6}$ part $\equiv$ 30 minutes (×6 to both sides)

$\frac{5}{6}\times 6 \equiv 30\times 6$ min

$\frac{5}{\cancel{6}1}\times \cancel{6}1 \equiv 180$ min

5 wholes $\equiv$ 180 min (÷5 for both sides)

$\frac{5}{5}$ whole $\equiv \frac{180}{5}$ min

$\frac{\cancel{5}1}{\cancel{5}1}$ whole $\equiv \frac{\cancel{180}36}{\cancel{5}1}$ min

1 whole $\equiv$ 36 min

$\therefore$ **The time when Gopal completes the walkathon is**

$H = 7.30$ a.m. $+ 36$ min

$= 8.06$ a.m. **(ans)**

(b) $\therefore$ Let v be Raju's average speed for the walkathon in m/min.

After Raju and Gopal have started,

- At the end of the 1st min, Raju is 20 m ahead of Gopal.
- At the end of the 2nd min, Raju is $2\times 20 = 40$ m ahead of Gopal.
- At the end of the 3rd min, Raju is $3\times 20 = 60$ m ahead of Gopal.
- At the end of the 4th min, Raju is $4\times 20 = 80$ m ahead of Gopal.
- At the end of the 30th min, Raju is $30\times 20 = 600$ m ahead of Gopal.

At this time, Gopal still have $1 - \frac{5}{6} = \frac{1}{6}$ part to go.

$\Rightarrow \frac{1}{6}$ part $\equiv$ 600 m

$\Rightarrow \frac{1}{6}\times 6$ parts $\equiv 600\times 6$ m

$\Rightarrow$ 1 whole $\equiv$ 3 600 m (total distance)

$\therefore$ **Raju's average speed for the walkathon in m/min is**

$v = \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{3600 \text{ m}}{30 \text{ min}}$

$= \frac{\cancel{3600}\ \cancel{360}120}{\cancel{30}\ \cancel{3}1}$

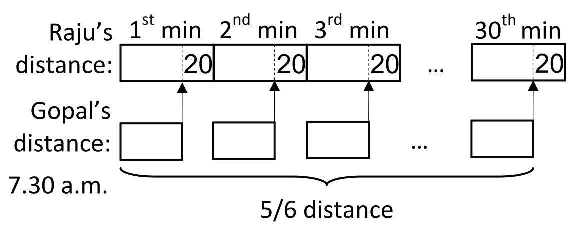
$= 120$ m/min **(ans)**

Approach II – model

In one school's walkathon, two competitors Raju and Gopal started at 7.30 a.m. Raju's average speed was 20 m/min faster than Gopal. By the time Raju completes the walkathon in 30 minutes, Gopal has only walked 5/6 of the distance.

(a) $\therefore$ Let H be the time when Gopal completes the walkathon.

The model based on the information given is as shown below.



In 30 minutes, Gopal has only completed $\frac{5}{6}$ of the distance.

$\Rightarrow$ In $\frac{1}{6}$ of the distance, Gopal have spent

$= \frac{30}{5}$

$= \frac{\cancel{30}\,6}{\cancel{5}\,1}$

$= 6$ min

$\Rightarrow$ At the full distance $\left(6 \times \frac{1}{6}\right)$, Gopal would have spent

$= 6 \times 6$ min

$= 36$ min

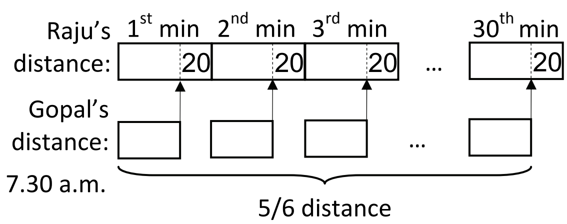
$\therefore$ **The time when Gopal completes the walkathon is**

$H = 7.30$ a.m. $+ 36$ min

$= 8.06$ a.m. **(ans)**

(b) $\therefore$ Let v be Raju's average speed for the walkathon in m/min.

The model based on the information given is as shown below.



After Raju and Gopal have started,

- At the end of the 1st min, Raju is 20 m ahead of Gopal.
- At the end of the 2nd min, Raju is $2 \times 20 = 40$ m ahead of Gopal.
- At the end of the 3rd min, Raju is $3 \times 20 = 60$ m ahead of Gopal.
- At the end of the 4th min, Raju is $4 \times 20 = 80$ m ahead of Gopal.
- At the end of the 30th min, Raju is $30 \times 20 = 600$ m ahead of Gopal.

  At this time, Gopal still have $1 - \frac{5}{6} = \frac{1}{6}$ part to go.

$\Rightarrow \frac{1}{6}$ part $\equiv 600$ m

$\Rightarrow \frac{1}{6} \times 6$ parts $\equiv 600 \times 6$ m

$\Rightarrow$ 1 whole $\equiv 3\,600$ m (total distance)

$\therefore$ **Raju's average speed for the walkathon in m/min is**

$v = \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{3600 \text{ m}}{30 \text{ min}}$

$= \frac{\cancel{3600}\ \cancel{360}\,120}{\cancel{30}\ \cancel{3}\,1}$

$= 120$ m/min **(ans)**

☺ **CheckBack**

(a) If the answer is 8.06 a.m.,

The time elapsed

$= 8.06$ a.m. – 7.30 a.m.

$= 36$ min

At constant average speed, at the 30th min, Gopal would have travelled

$= \frac{\text{time elapsed}}{\text{total time taken}}$

$= \frac{30 \text{ min}}{36 \text{ min}} = \frac{\cancel{30}\,5}{\cancel{36}\,6}$

$= 5/6$ of the distance **(checked)**

(b) If the answer is 120 m/min (Raju's speed),

Gopal's speed $= \frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{3600 \text{ m}}{36 \text{ min}} = \frac{\cancel{3600}\,100}{\cancel{36}\,1}$

$= 100$ m/min

$\therefore$ Raju's average speed was 20 m/min faster than Gopal.

(checked)

☺ **Exam Report**

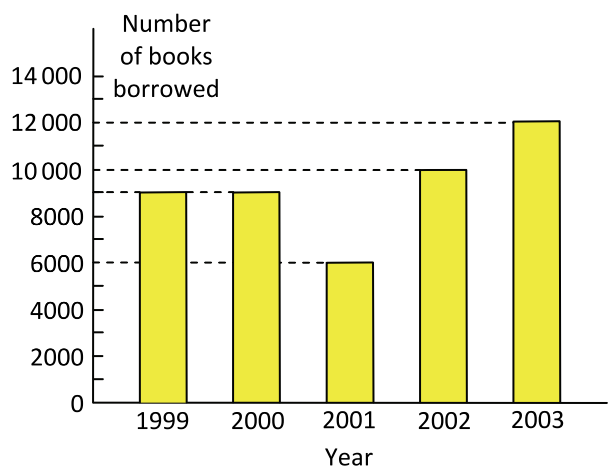
This question presented little problem to many candidates. For those candidates that did not give the correct answer, it was frequently due to problem involved in interpreting the question itself.

3. [Speed] [2001–2004]

Method

Approach I – data analysis

A library tabulated a bar graph to show the number of books borrowed each year from 1999 to 2003.



(a) ∴ Let *Y* be the 1–year period in which there was the biggest change in the number of books borrowed.

Re–tabulate the bar graph into a table:

Year	Books borrowed	Change (year on year)
1999	9000	
2000	9000	+0
2001	6000	–3000
2002	10000	+4000
2003	12000	+2000

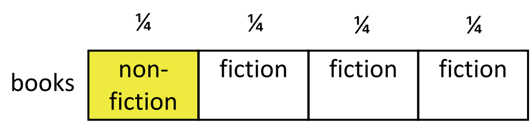
∴ **The 1–year period in which there was the biggest change in the number of books borrowed is**

Between year 2001 **and** 2002 (*Y*). **(ans)**

(b) In the year 2000, a librarian noticed that for every non-fiction book borrowed, 3 fiction books were borrowed.

∴ Let *M* be the number of more fiction books than non–fiction books were borrowed in 2000.

The total number of books borrowed in year 2000 is 9000. Drawing the model based on the information provided:



∴ **The number of more fiction books than non–fiction books were borrowed in 2000 is**

$M = \text{No}_{\text{fiction books}} - \text{No}_{\text{non–fiction books}}$

$= \frac{2}{4} \times 9000$

$= ½ \times 9000$

$= 4\,500$ books **(ans)**

(c) There was an average of 9500 books borrowed from year 1998 to year 2003.

∴ Let *B* be the number of books borrowed in 1998.

The total number of books borrowed from year 1998 to 2003

$= 9500 \times 6$

$= 57\,000$

∴ The number of books borrowed in 1998 is

$B = 57000 - 9000 - 9000 - 6000 - 10000 - 12000$

$= 11\,000$ **(ans)**

☺ **CheckBack**

(a) If the answer is between year 2001 and 2002,

Between the year 2001 and year 2002, the bars have the greatest difference in length. **(checked)**

(b) If the answer is 4500 books,

$B_f - B_{nf} = 4500$ —— ❶

$B_f + B_{nf} = 9000$ —— ❷

❶+❷:

$B_f - B_{nf} + B_f - B_{nf} = 4500 + 9000$

$2\times B_f = 13500$ (÷2 for both sides)

$2\times B_f \div 2 = 13500 \div 2$

$\Rightarrow B_f = 6750$

$\Rightarrow B_{nf} = 9000 - 6750 = 2250$

Ratio $B_{nf} : B_f$

$\equiv 2250 : 6750 \equiv \cancel{2250}\,1 : \cancel{6750}\,3$

$\equiv 1 : 3$

(checked)

☺ **Exam Report**

This question presented little problem to many candidates.

End of 2001–2005

Learning objectives implemented in the period:

Start of 2005–2009

4. [Speed] [2006–2009]

Method

Approach I – rate

At 0900h, a lorry starts from town X and travels towards town Y at a constant speed of 55 km/h. Two hours later, at 1100h, a car starts from town Y towards town X, and travels at a constant speed, until it passes the lorry at 1300h. At this point, the lorry has only travelled $\frac{5}{9}$ of the journey. When the car passes the lorry, the car decreases its speed by 8 km/h and travels at new constant speed to complete the journey.

∴ Let T be the time the car reaches town X.

At 1300h, the distance travelled by the lorry

$= \text{speed (in km/h)} \times \text{time taken (h)}$

$= 55\text{ km/h} \times (1300\text{h} - 0900\text{h})$

$= 55\text{ km/h} \times 4\text{ h}$

$= 220\text{ km}$

$\Rightarrow \frac{5}{9}$ of the journey $\equiv 220$ km (×9 to both sides)

$\frac{5}{9}$ of the journey $\times 9 \equiv 220\text{ km} \times 9$

$5 \times$ the journey $\equiv 1\,980$ km (÷5 to both sides)

$5 \times$ the journey $\div 5 \equiv 1\,980\text{ km} \div 5$

$\Rightarrow$ the journey $\equiv \frac{1\,980\text{ km}}{9}$

$= \frac{\cancel{1\,980}\ 396\text{ km}}{\cancel{5}\ 1}$

$= 396$ km

At 1300h, the distance travelled by the car

$= \text{total distance} - \text{lorry's distance}$

$= 396 \text{ km} - 220 \text{ km}$

$= 176 \text{ km}$

Speed of the car for the first part of the journey

$= \text{distance travelled (km)} \div \text{time taken (h)}$

$= 176 \text{ km} \div (1300\text{h} - 1100\text{h})$

$= 176 \text{ km} \div 2 \text{ h}$

$= 88 \text{ km/h}$

Speed of the car for the second part of the journey

$= 88 \text{ km/h} - 8 \text{ km/h}$

$= 80 \text{ km/h}$

Time taken for second part of the car's journey

$= \text{distance travelled (km)} \div \text{speed (km/h)}$

$= 220 \text{ km} \div 80 \text{ km/h}$

$= \frac{220}{80} \text{ h}$

$= \frac{\cancel{220}\ \cancel{22}\ 11}{\cancel{80}\ \cancel{8}\ 4} \text{ h}$

$= \frac{11}{4} \text{ h}$

$= 2\frac{3}{4} \text{ h}$

$= 2 \text{ h } 45 \text{ mins}$

∴ The time the car reaches town X is

$T = 1300\text{h} + 2 \text{ h } 45 \text{ mins}$

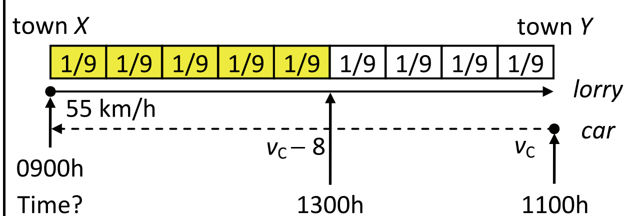
$= 1545\text{h}$ **(ans)**

Approach II – model

At 0900h, a lorry starts from town X and travels towards town Y at a constant speed of 55 km/h. Two hours later, at 1100h, a car starts from town Y towards town X, and travels at a constant speed, until it passes the lorry at 1300h. At this point, the lorry has only travelled $\frac{5}{9}$ of the journey. When the car passes the lorry, the car decreases its speed by 8 km/h and travels at new constant speed to complete the journey.

∴ Let T be the time the car reaches town X.

The model based on the information given is as shown below.



At 1300h, the distance travelled by the lorry

$= \text{speed (in km/h)} \times \text{time taken (h)}$

$= 55 \text{ km/h} \times (1300\text{h} - 0900\text{h})$

$= 55 \text{ km/h} \times 4 \text{ h}$

$= 220 \text{ km}$

⇒ 5 pieces of 1/9–part ≡ 220 km

1 piece of 1/9–part ≡ 220 km ÷ 5

$= \frac{220 \text{ km}}{5}$

$= \frac{\cancel{220}\ 44 \text{ km}}{\cancel{5}\ 1}$

⇒ The total distance (X →Y)

≡ 9 pieces of 1/9–part

$= 9 \times 44 \text{ km}$

$= 396 \text{ km}$

At 1300h, the distance travelled by the car

$= 4$ pieces of 1/9–part

$= 4 \times 44$ km

$= 176$ km

Speed of the car for the first part of the journey, v_C

$=$ distance travelled (km) $\div$ time taken (h)

$= 176$ km $\div$ (1300h – 1100h)

$= 176$ km $\div$ 2 h

$= 88$ km/h

Speed of the car for the second part of the journey, $v_C - 8$

$= 88$ km/h – 8 km/h

$= 80$ km/h

Time taken for second part of the car's journey

$=$ distance travelled (km) $\div$ speed (km/h)

$= 220$ km $\div$ 80 km/h

$= \frac{220}{80}$ h

$= \frac{\cancel{220}\ \cancel{22}\ 11}{\cancel{80}\ \cancel{8}\ 4}$ h

$= \frac{11}{4}$ h

$= 2\frac{3}{4}$ h

$= 2$ h 45 mins

∴ The time the car reaches town X is

$T = 1300\text{h} + 2$ h 45 mins

$= 1545\text{h}$ **(ans)**

☺ **CheckBack**

There is no easy CheckBack option for this question. There are too many pre–condition data.

☺ **Exam Report**

The question was considered difficult as very few scripts gave the correct answer. In most scripts, the question was unanswered.

Candidates should note that working marks might still be granted if the initial model or algebra was correct, even though the final answer was not.

Candidates who used the model approach fared much better than those who used other approaches.

5. [Speed] [2006–2009]

Method

Approach I – rate

At different constant speeds, David and Michael drive from town A to town B. David starts his journey 30 minutes earlier than Michael. But, ended up, Michael reaches town B 50 minutes earlier than David. When Michael reaches town B, David has travelled $\frac{4}{5}$ of the journey and is still 75 km away from town B.

(a) ∴ Let D be the total distance between town A and town B.

Deduce that,

1/5 of the total distance $\equiv 75$ km

∴ The total distance between town A and town B is

$D \equiv 5 \times 75$ km

$= 375$ km **(ans)**

(b) $\therefore$ Let v_D be the number of kilometres David travels in 1 hour.

"Michael reaches town B 50 minutes earlier than David. When Michael reaches town B, David has travelled $\frac{4}{5}$ of the journey and is still 75 km away from town B."

David therefore will travel 75 km in 50 min or $\frac{50}{60}$ h.

$\therefore$ **The number of kilometres David travels in 1 hour is**

v_D = distance travelled (km) ÷ time taken (h)

$= 75 \text{ km} \div \frac{50}{60} \text{ h}$

$= 75 \text{ km} \times \frac{60}{50} \text{ h}$

$= 75 \times \frac{60}{50}$

$= \cancel{75}\,15 \times \frac{\cancel{60}\,6}{\cancel{50}\,\cancel{5}\,1}$

$= 15 \times 6$

$= 90$ km per hour **(ans)**

(c) $\therefore$ Let T be the time taken by Michael to travel from town A to town B.

Total time, David needs to take

= total distance travelled (km) ÷ speed (km/h)

$= 375 \text{ km} \div 90 \text{ km/h}$

$= \frac{375}{90}$ h

$= \frac{\cancel{375}\,\cancel{75}\,25}{\cancel{90}\,\cancel{18}\,6}$ h

$= \frac{25}{6} \times 60$ min

$= \frac{25}{\cancel{6}\,1} \times \cancel{60}\,10$ min

$= 25 \times 10$ min

$= 250$ min

$\therefore$ **The time taken by Michael to travel from town A to town B is**

$T = 250 \text{ min} - 30 \text{ min} - 50 \text{ min}$

$= 170$ min

$= 120 \text{ min} + 50 \text{ min}$

$= 2 \times 60 \text{ min} + 50 \text{ min}$

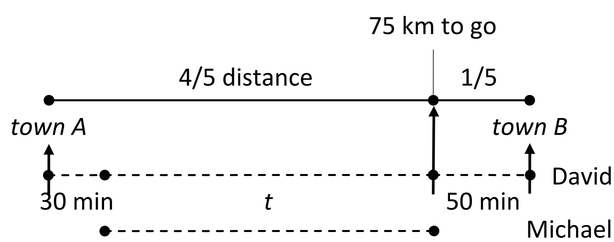
$= 2 \text{ h} + 50 \text{ min}$

$= 2$ h 50 min **(ans)**

Approach II – model

At different constant speeds, David and Michael drive from town A to town B. David starts his journey 30 minutes earlier than Michael. But, ended up, Michael reaches town B 50 minutes earlier than David. When Michael reaches town B, David has travelled $\frac{4}{5}$ of the journey and is still 75 km away from town B.

The model based on the information given is as shown below.



(a) $\therefore$ Let D be the total distance between town A and town B.

Deduce that,

1/5 of the total distance ≡ 75 km

∴ **The total distance between town A and town B is**

$D \equiv 5 \times 75$ km

$= 375$ km **(ans)**

(b) ∴ Let v_D be the number of kilometres David travels in 1 hour.

From the model, David needs to travel another 75 km in 50 min or $\frac{50}{60}$ h to reach town B.

∴ **The number of kilometres David travels in 1 hour is**

v_D = distance travelled (km) ÷ time taken (h)

$= 75 \text{ km} \div \frac{50}{60} \text{ h}$

$= 75 \text{ km} \times \frac{60}{50} \text{ h}$

$= 75 \times \frac{60}{50}$

$= \cancel{75}\,15 \times \frac{\cancel{60}\,6}{\cancel{50}\,\cancel{5}\,1}$

$= 15 \times 6$

$= 90$ km per hour **(ans)**

(c) ∴ Let t be the time taken by Michael to travel from town A to town B.

Total time, David needs to take

= total distance travelled (km) ÷ speed (km/h)

$= 375 \text{ km} \div 90 \text{ km/h}$

$= \frac{375}{90}$ h

$= \frac{\cancel{375}\,\cancel{75}\,25}{\cancel{90}\,\cancel{18}\,6}$ h

$= \frac{25}{6} \times 60$ min

$= \frac{25}{\cancel{6}\,1} \times \cancel{60}\,10$ min

$= 25 \times 10$ min

$= 250$ min

∴ **The time taken by Michael to travel from town A to town B is**

$t = 250$ min – 30 min – 50 min

$= 170$ min

$= 120$ min $+ 50$ min

$= 2 \times 60$ min $+ 50$ min

$= 2$ h $+ 50$ min

$= 2$ h 50 min **(ans)**

☺ **CheckBack**

(a) If the answer is 375 km,

- 1/5 of the distance = 375 km ÷ 5

  = 75 km **(condition)**

(b) If the answer is 90 km/h,

- David's time of travel for 75 km

  = distance (km) ÷ speed (km/h)

  $= 75 \text{ km} \div 90 \text{ km/h} = \frac{75}{90}$ h

  $= \frac{\cancel{75}\,5}{\cancel{90}\,6}$ h $= \frac{5}{6}$ h

  = 50 min **(condition)**

(c) If the answer is 2 h 50 min,

- David's total time taken

  = distance (km) ÷ speed (km/h)

  $= 375 \text{ km} \div 90 \text{ km/h}$

  $= \frac{375}{90}$ h $= \frac{\cancel{375}\,25}{\cancel{90}\,6}$ h

  $= \frac{25}{6}$ h $= 4\frac{1}{6}$ h

- Using the answer, David's total

time taken

$= 30$ m + 2 h 50 m + 50 m

$= 2$ h + 130 min

$= 2$ h + 120 min + 10 min

$= 2$ h + 2×60 min + 10 min

$= 4$ h 10 min **(condition)**

(checked)

This question–type is considered difficult as it involves all three very different groups of variables (unknowns) – distances, speeds and time. All of these unknowns are fully standalone and yet interlinked with the other unknowns. In this particular case, time is the most important central model in resolving the question, where typically the drawing of the model based on distances would have sufficed.

Rote learning is currently the only approach to successfully score in the final examination.

☺ **Exam Report**

The question was considered difficult as very few scripts gave the correct answer. In most scripts, the question was unanswered. Candidates who managed to arrive at the correct answers, managed to follow the question parts provided. This method of answering provided the critical procedure needed to resolve the problem sum step–by–step.

Candidates should note that working marks might still be granted if the initial model or algebra was correct, even though the final answer was not.

Candidates who used the model approach fared much better than those who used other approaches.

6. [Speed] [2006–2009]

Method

Approach I – rate

Alice and Bobby drive at constant speeds. At 10.15 a.m., Alice leaves town X for town Y driving at a speed of 80 km/h. Then at 11.15 a.m., Bobby also leaves town X for town Y driving at a certain speed. At 1.15 p.m., both of them pass a petrol station that is away 100 km away from town Y.

(a) ∴ Let v_B be Bobby's driving speed.

At the petrol station, total time elapsed for Alice

$= 1.15$ p.m. – 10.15 a.m.

$= 3$ h

At Alice's 80 km/h, total distance from town X to petrol station

= speed (km/h) × time taken (h)

$= 80$ km/h × 3 h

$= 240$ km

At the petrol station, total time elapsed for Bobby

$= 1.15$ p.m. – 11.15 a.m.

$= 2$ h

∴ **Bobby's driving speed is**

v_B = total distance (km) ÷ total time elapsed (h)

$= 240$ km ÷ 2 h

$= 120$ km/h **(ans)**

(b) ∴ Let M be the number of minutes Bobby reaches town Y earlier than Alice.

From the petrol station,

- Alice's time to reach town Y

  = distance (km) ÷ speed (km/h)

$= 100 \text{ km} \div 80 \text{ km/h}$

$= \dfrac{100}{80}$ h

$= \dfrac{\cancel{100}\ \cancel{10}\ 5}{\cancel{80}\ \cancel{8}\ 4}$ h

$= \dfrac{5}{4}$ h

$= 1\frac{1}{4}$ h

$=$ 1 h 15 min

- Booby's time to reach town Y

$=$ distance (km) $\div$ speed (km/h)

$= 100 \text{ km} \div 120 \text{ km/h}$

$= \dfrac{100}{120}$ h

$= \dfrac{\cancel{100}\ \cancel{10}\ 5}{\cancel{120}\ \cancel{12}\ 6}$ h

$= \dfrac{5}{6}$ h

$=$ 50 min

$\therefore$ **The number of minutes Bobby reaches town Y earlier than Alice is**

$M =$ 1 h 15 min – 50 min

$=$ 1 h + 15 min – 50 min

$=$ 60 min + 15 min – 50 min

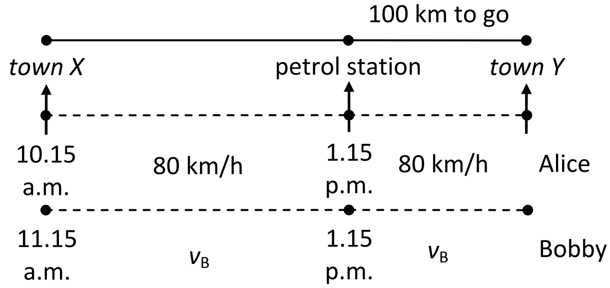
$=$ 75 min – 50 min

$=$ 25 min **(ans)**

Approach II – model

Alice and Bobby drive at constant speeds. At 10.15 a.m., Alice leaves town X for town Y driving at a speed of 80 km/h. Then at 11.15 a.m., Bobby also leaves town X for town Y driving at a certain speed. At 1.15 p.m., both of them pass a petrol station that is away 100 km away from town Y.

The model based on the information given is as shown below.



(a) $\therefore$ Let v_B be Bobby's driving speed.

At the petrol station, total time elapsed for Alice

$=$ 1.15 p.m. – 10.15 a.m.

$=$ 3 h

At Alice's 80 km/h, total distance from town X to petrol station

$=$ speed (km/h) $\times$ time taken (h)

$= 80 \text{ km/h} \times 3 \text{ h}$

$=$ 240 km

At the petrol station, total time elapsed for Bobby

$=$ 1.15 p.m. – 11.15 a.m.

$=$ 2 h

$\therefore$ **Bobby's driving speed is**

$v_B =$ total distance (km) $\div$ total time elapsed (h)

$= 240 \text{ km} \div 2 \text{ h}$

$=$ 120 km/h **(ans)**

(b) $\therefore$ Let M be the number of minutes Bobby reaches town Y earlier than Alice.

From the petrol station,

- Alice's time to reach town Y

$=$ distance (km) $\div$ speed (km/h)

$= 100 \text{ km} \div 80 \text{ km/h}$

$= \dfrac{100}{80}$ h

$= \frac{\cancel{100}\ \cancel{10}\ 5}{\cancel{80}\ \cancel{8}\ 4}$ h

$= \frac{5}{4}$ h

$= 1\frac{1}{4}$ h

$=$ 1 h 15 min

- Booby's time to reach town Y

  $=$ distance (km) $\div$ speed (km/h)

  $=$ 100 km $\div$ 120 km/h

  $= \frac{100}{120}$ h

  $= \frac{\cancel{100}\ \cancel{10}\ 5}{\cancel{120}\ \cancel{12}\ 6}$ h

  $= \frac{5}{6}$ h

  $=$ 50 min

∴ **The number of minutes Bobby reaches town Y earlier than Alice is**

$M =$ 1 h 15 min – 50 min

$=$ 1 h $+$ 15 min – 50 min

$=$ 60 min $+$ 15 min – 50 min

$=$ 75 min – 50 min

$=$ 25 min **(ans)**

☺ **CheckBack**

(a) If the answer is 120 km/h,

- Total distance from town X to petrol station

  $=$ speed (km/h)$\times$time taken (h)

  $=$ 120 km/h $\times$ 2 h

  $=$ 240 km

- Alice's speed

  $=$ distance (km) $\div$ time taken (h)

  $=$ 240 km $\div$ 3 h

  $=$ 80 km/h **(condition)**

(b) There is no easy CheckBack option for this question part. The pre–condition data were too many. **(checked)**

☺ **Exam Report**

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Candidates should note that working marks might still be granted if the initial model or algebra was correct, even though the final answer was not.

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End of 2005–2009