



$$x = Ct^3 \text{ and } v = \frac{dx}{dt} = \frac{d(Ct^3)}{dt} = 3Ct^2$$

Find the velocity by applying the definition formula.

$$a = \frac{dv}{dt} = \frac{d(3Ct^2)}{dt} = 6Ct \quad (\text{ans})$$



Example 11

A student throws a ball upwards at an initial speed of 9.81m/s.

- How long does it take to reach the highest point?
- What is the distance to the highest point?
- What is the total time the ball is in the air?

Solution:

- When the ball reaches the highest point, the velocity is 0.

$$v = v_0 + at, \text{ so } t = \frac{v - v_0}{a} = \frac{9.81 - 0}{-9.81} = 1 \text{ s} \quad (\text{ans})$$

- The distance travelled to the highest point:

$$\Delta x = v_0 t + \frac{1}{2} at^2 = 9.81(1) + \frac{1}{2}(-9.81)(1)^2 = 4.91 \text{ m} \quad (\text{ans})$$

- The time for the ball to travel back to the student:

$$x_f - x_i = v_0 + \frac{1}{2} at_{total}^2 = 0$$

$$t_{total} = \sqrt{-2 \frac{v_0}{a}} = \sqrt{\frac{(-2)(9.81)}{(-9.81)}} = 1.41 \text{ s} \quad (\text{ans})$$

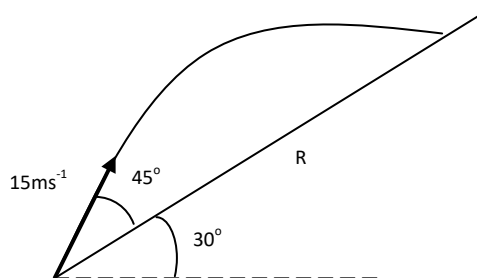


2.2 Non-linear motion

fundamental questions

Example 1

A projectile is fired uphill over a slope which is inclined at an angle of 30° to the horizontal. The launch velocity of the projectile is 15 m s^{-1} at an angle of 45° to the horizontal and the projectile impacts on the slope at a distance R along the slope from the point of the launch.



- By considering the horizontal motion of the projectile, express the time taken t , in terms of R , for the projectile to impact on the slope. [3]
- By considering the vertical motion of the projectile, express R in terms of t . [2]
- Hence calculate the value of R . [2]

Solution:

- Projectile to impact

$$U_x = 15 \cos 45^\circ \Rightarrow S_x = R \cos 30^\circ = U_x t$$

$$t = \frac{S_x}{U_x} = \frac{R \cos 30^\circ}{15 \cos 45^\circ} = 0.0816R \quad (\text{ans})$$

- Expression for projectile

$$\text{Since } U_y = 15 \sin 45^\circ, S_y = R \sin 30^\circ, a_y = -9.81 \text{ m s}^{-2}$$

$$\text{Using } s_y = u_y t + \frac{1}{2} a_y t^2, \therefore R \sin 30^\circ = 15 \sin 45^\circ t + \frac{1}{2} (-9.81) t^2$$

$$R = 15\sqrt{2} t - 9.81 t^2 = 21.2t - 9.81 t^2 \quad (\text{ans}) \quad \textcircled{1}$$

- Substitute $t = 0.0816 R$ into $\textcircled{1}$

$$R = 21.2(0.0816R) - 9.81(0.0816R)^2$$

$$0.0654R^2 - 0.732R = 0$$

$$R = 0 \text{ (NA)} \quad R = 11.2 \text{ m} \quad (\text{ans})$$

Mark Scheme:

$$(i) \quad U_x = 15 \cos 45^\circ$$

B1

$$S_x = R \cos 30^\circ$$

B1

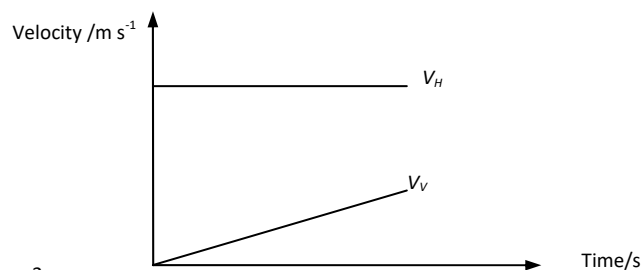


Figure 2

- (b) Figure 2 show the variation with time of V_H , the horizontal component of the velocity and V_V , the vertical component of the velocity of the bullet.

On the same axes on Fig 2 sketch the graphs showing how V_H and V_V vary with time when air resistance is taken into account. Label the graph H and V respectively. Briefly explain your answers. [5]

- (c) Hence or otherwise, explain whether it takes longer or shorter time to reach the ground. [1]

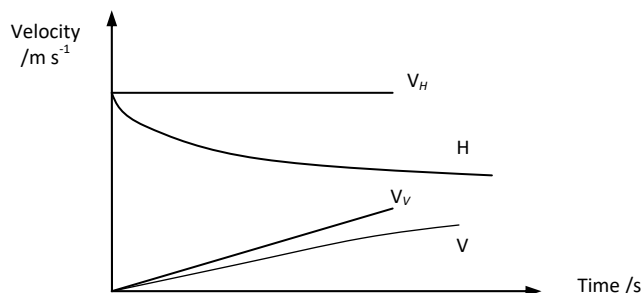
Solution:

- (a) Vertically, take downwards as positive

$$v^2 = u^2 + 2as = 0 + 2(9.81)(13.5)$$

$$v = 16.27 \text{ m s}^{-1} \text{ (ans)}$$

- (b)



With the effect of air resistance, there will be opposing force acting on the object that is against motion.

For the horizontal component of the velocity;

It decelerates horizontally. As the horizontal velocity decreases, this opposing force will also decrease and hence the deceleration (gradient of the graph) decreases with time.

For the vertical component of the velocity;

With the effect of air resistance, the vertical acceleration will decrease with increasing velocity and hence the gradient decreases with time. (ans)

- (c) With reference to the graph, to travel the same vertical distance, which is equal to the area under the v - t graph, it will take a longer time to reach the ground. (ans)

Mark Scheme:

- | | | |
|-----|--|---------|
| (a) | Take downwards as positive | B1 |
| | $v^2 = u^2 + 2as$ | B1 |
| | $v^2 = 0 + 2(9.81)(13.5)$ | B1 |
| | $v = 16.28 \text{ m s}^{-1}$ | B1 |
| (b) | Graphs are drawn clearly | B1 |
| | Graphs are labeled | B1 |
| | Air resistance, there will be opposing force acting on the object against motion | B1 |
| | Horizontal velocity decreases, opposing force decreases | |
| | and deceleration decreases with time | B1 |
| | Vertical acceleration decreases with increasing velocity, | |
| | gradient decreases with time | B1 |
| (c) | Explained clearly | B1 [10] |



Example 4

A student throws a ball into the air with an initial velocity of 20m/s at 30° from the horizontal. Find

- (a) the total time the ball is in the air
- (b) the total horizontal distance travelled.

Solution:

- (a) When the ball travels back, the vertical displacement is 0

Therefore,

$$y = v_{0y}t - \frac{1}{2}gt^2 = 0;$$

$$t = \frac{2v_{0y}}{g} = \frac{2v_0 \sin \theta}{g} = \frac{2(20)\sin 30^\circ}{9.81} = 2 \text{ s} \quad (\text{ans})$$

- (b) In the horizontal direction, there is no acceleration, therefore, the velocity is constant.

$$x = v_{0x}t = (v_0 \cos \theta)t = (20)(\cos 30^\circ)(2) = 34.6 \text{ m} \quad (\text{ans})$$



Example 5

A helicopter drops a supply package to soldier in a jungle. When the package is dropped, the helicopter is 100 m above the ground and flying at 25 m/s. Where does the package land?

Solution:

When the package hits the ground, the vertical displacement is 100

$$y = v_{0y}t + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(100)}{9.81}} = 4.5 \text{ s}$$

$$\text{Horizontal displacement is } x = v_{0x}t = (25)(4.5) = 112.5 \text{ m} \quad (\text{ans})$$



**Solution:**

- (a) The work done by the pulling force is the product of the force and the distance travelled along the incline :

$$W_f = F \cdot S = (70)(12) = 840 \text{ J} \quad (\text{ans})$$

- (b) Potential energy of the cart at the top is the product of the weight of the goods and the elevation h:

$$mgh = (30)(9.81)(1) = 294.3 \text{ J} \quad (\text{ans})$$

- (c) By definition, the work done by the gravity is the same as the change in the potential energy mgh

$$W_g = mgh = (30)(9.81)(1) = 294.3 \text{ J} \quad (\text{ans})$$

**Example 3**

A box of 200 kg is to be loaded to truck by a crane that exerts an upward force of 2000 N on the box. This force is constantly applied and lifted up the box over a distance of 1 m. Find

- (a) the work done by the crane. [1]
(b) the upward speed of the box after 2 m. [2]

Solution:

- (a) The work done by the crane:

$$W_{\text{crane}} = F_{\text{crane}} \Delta x = (2000\text{N})(1\text{m}) = 2000 \text{ kJ} \quad (\text{ans})$$

- (b) The final speed is related to the final kinetic energy:

$$K_f = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2K_f}{m}}$$

Apply the work-kinetic energy theorem, with $v_i = 0$

$$W_{\text{total}} = K_f - K_i = K_f$$

The total work is the sum of the applied work by the crane and the work done by gravity

$$W_{\text{total}} = W_{\text{crane}} + W_g = 2000 - 200 \times 9.81 = 38 \text{ J}$$

$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(38 \text{ J})}{200\text{kg}}} = 0.6 \text{ m s}^{-1} \quad (\text{ans})$$

Mark Scheme:

- (a) Correct final numerical answer
(b) Apply formula $W_{\text{total}} = W_{\text{crane}} + W_g$
Correct final numerical answer

A1**M1****A1****[3]**