

# Solving quadratic equations by factorisation

1 • 1

## Questions

- Factorise completely  $48c^2 - 27d^2$ .
- Factorise  $x^6 - 3x^3 + 2$ .
- Factorise  $15ax + 21bx - 14by - 10ay$  completely.
- Factorise completely
  - $8a - 4a^2$
  - $25y^2 - 4$
  - $2x^2 + 6xz + 5xy + 15yz$
- Factorise completely the following expressions:
  - $6pq^3 - 14p^3q^2$
  - $3kx - ky + 9x - 3y$
- Factorise completely  $x^2 - 3x + 9y - 3xy$ .
- Factorise completely
  - $8q^2 - 50p^2$
  - $m^2 - 2m - 3mn + 6n$
- Factorise completely
  - $6a^2 + 3ab - 8ka - 4kb$
  - $5p^3 - 125p$
- Factorise completely
  - $16x^2 - 4y^2$
  - $1 - x - y + xy$
- Factorise  $6bx - 2by + ay - 3ax$ .
- Factorise completely:
  - $8x^2 - 18xy + 4y^2$
  - $5a^2b + 20ab^2 - a - 4b$
- Factorise completely
  - $3r(p - 5q) + (5q - p)$
  - $4x^2 + 4x + 1 - 9y^2$
- Factorise  $2w^2 + w - 15$ .
  - Hence or otherwise solve  $2(x - 1)^2 + x - 16 = 0$ .
- Factorise  $t^4 - 16$  completely.
  - If  $(x + \frac{2}{x})^2 = 36$ , find the value of  $(x^2 + \frac{4}{x^2})$ .
- Factorise  $p^2q^2 + 9pq + 3apq + 27a$ .
  - Find the value of  $x + y + z$ , given that  $\frac{x}{3-x} = \frac{y}{5-y} = \frac{z}{16-z} = 2$ .
- Factorise completely  $3ay - 3by - a^2 + b^2$
  - Express  $\frac{5}{x-2} - \frac{4}{x-5} + \frac{1}{2-x}$  as a single fraction in its simplest form.
- Factorise  $ka + kb$ , and hence find the value of  $2.58 \times 32.5 + 67.5 \times 2.58$ .
  - If  $x = 3$  is a solution of  $x^2 + 8x + k = 0$ , find the value of  $k$ , hence find the other value of  $x$ .
  - Factorise  $(y + z)^4 - 13(y + z)^2 + 36$  completely.

**Answer keys:**

●●●

1.  $3(4c-3d)(4c+3d)$
2.  $(x^3-1)(x^3-2)$
3.  $(3x-2y)(5a+7b)$
4. (a)  $4a(2-a)$   
(b)  $(5y+2)(5y-2)$   
(c)  $(2x+3y)(x+3z)$
5. (a)  $2pq^2(3q-7p^2)$   
(b)  $(3x-y)(k+3)$
6.  $(x-3)(x-3y)$
7. (a)  $2(2q-5p)(2q+5p)$   
(b)  $(m-3n)(m-2)$
8. (a)  $(2a+b)(3a-4k)$   
(b)  $5p(p-5)(p+5)$
9. (a)  $4(2x+y)(2x-y)$   
(b)  $(1-x)(1-y)$
10.  $(2b-a)(3x-y)$
11. (a)  $2(4x-y)(x-2y)$   
(b)  $(a+4b)(5ab-1)$
12. (a)  $(p-5q)(3r-1)$   
(b)  $(2x+1+3y)(2x+1-3y)$
13. (a)  $(2w-5)(w+3)$   
(b)  $x=3\frac{1}{2}$  or  $x=-2$
14. (a)  $(t-2)(t+2)(t^2+4)$   
(b) 32
15. (a)  $(pq+3a)(pq+9)$   
(b) 16
16. (a)  $(a-b)(3y-a-b)$   
(b)  $\frac{12}{(2-x)(x-5)}$
17. (a)  $k(a+b)$ , 258  
(b)  $k=-33$ ,  $x=-11$   
(c)  $\frac{[(y+z-2)(y+z+2)]}{(y+z-3)(y+z+3)}$
18. (a)  $(x-2y)(2x^2-y)$   
(b)  $d=-8\frac{2}{3}$  or  $d=0$
19. (a)  $(y+3)(x+4)$   
(b)  $(y^2+2)(2y-1)(2y+1)$
20.  $3(x-y)^2$
21. (a)  $(a+3b)(2x-1)(2x+1)$   
(b)  $x(2y-\frac{3}{4}z)(2y+\frac{3}{4}z)$
22.  $(4-n)(m+1)^2$
23. (a)  $(3-q)(3-p)$   
(b)  $(3x-2)(2x+5)$
24.  $(a+b)(a-b+1)$
25. (a)  $a(2b+3c)$   
(b)  $(x^2-1)(x-4)$   
(c)  $(8y-1)(y+1)$
26. (a)  $(x-9)(x-16)$   
(b)  $y=\pm 3$  or  $y=\pm 4$
27.  $(2x-5)(x-y)$
28. (a)  $2(3p+q)(3p-q)$   
(b)  $(3a-2b)(2x+1)$
29. (a)  $(3x+2y)(x-6y)$   
(b)  $(2q-5)(3p-2)$
30. (a)  $(x^2+1)(1+y)(1-y)$   
(b)  $\frac{5x+6}{(x+3)(x-1)}$
31. (a)  $3x(1-6x)$   
(b)  $(x+7)(x-2)$
32. (a)(i)  $4(p+3)(p-3)$   
(a)(ii)  $\frac{4(p-3)}{5p-7}$   
(b)  $-5x-8$
33. (a)  $(p+3r)(4p-r)$   
(b)  $\frac{y}{4x}$
34. (a) 3555546  
(b)  $(p-q)(p+q)(x-y)$
35.  $x(x+6)(x-6)$

**Answer keys:****1 • 2**

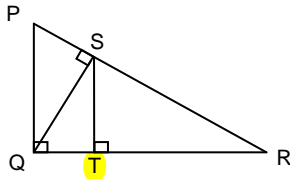
1.  $a^2 + 9ab - 6b^2$
2. (a)  $7 - 11x$   
(b)  $13xy - 4x^2 - 3y^2$
3.  $\frac{21}{25}$
4. (a)  $(x+2y)^2 - 9$   
(b)  $(2a-6b)(a+b)$   
(c)  $n = 2\frac{2}{3}, 2\frac{1}{2}$
5. (a)  $-x^2 + 11x - 32$   
(b)  $(3-m)(x-y)$   
(c)  $y = -1$  or  $y = 11$
6. 0
7. (a)  $x = -4$  or  $x = 5$   
(b)  $\frac{13x+14}{(x-1)(5x+4)}$
8.  $\frac{1}{3}$
9.  $x = 0.786$  or  $x = -2.12$
10.  $x = 1$  or  $x = -1$
11.  $x = -\frac{1}{3}$  or  $x = 4$
12. (a) 33  
(b)  $x = -6$  or  $x = 4$
13.  $2mn = 4$ ,  $(m-n)^2 = 8$
14. (a) 2 or 4  
(b)  $x = \frac{10}{9} = 1\frac{1}{9}$  or  $x = 5$
15.  $x = 10.82$  or  $x = -18.49$
16. (a)  $y = -\frac{3}{2}$  or  $y = 3.5$   
(b)  $x = 2.5$  or  $x = 2$
17.  $x = -0.586$  (3sf) or  $x = -3.41$  (3sf)
18. -0.71 or 4.51
19.  $x = 4$  or  $x = -3$
20. (a)  $x = 16$  or  $x = 8$   
(b)  $y = 4$  or  $y = 3$
21. 2.12 or -1.33
22. (a)  $x = 0$  or  $x = \frac{3}{4}$   
(b)  $x = \pm 3.32$
23. 1.14 or -1.47
24.  $x = 4$  or  $x = -1$
25. (a)  $x = -3$   
(b)  $x = 7$
26. -nil-
27. (a)  $x = 2$  or  $x = -5$   
(b)  $2\frac{1}{3}$
28.  $x = 4$  or  $x = -3$
29. (a)  $x = 4$  or  $x = -3$   
(b)  $p = 1$ ,  $q = 2$
30. (a)  $x = 7$ ,  $y = 20$   
(b)  $a = 2$ ,  $b = 1.5$
31. (a)  $k = 4$   
(b)  $x = -2$
32.  $a = 5$  or  $a = -1$
33. (a)  $a = \frac{3}{4}$  or  $a = 0$   
(b)  $y = \pm\frac{1}{2}$
34.  $x = 2.9, -1.9$
35.  $x = 2\frac{1}{3}$  or  $x = \frac{1}{2}$
36. (a)  $x = 0$  or  $x = 14$   
(b)  $(13+5p)(13-5p)$
37.  $x = 9$  or  $x = 6$   
 $x = 1.43$  or  $x = 0.543$
38. (a)  $(x-13)(x+12)$   
Two integers are 13 and 14  
(b) 25
39. (b)  $x = 0.237$  or  $x = -1.40$   
(c)  $0.507 \text{ m}^2$
40. 6.60 and 0.40
41.  $(x-1)^2 - 4$
42.  $(x-\frac{5}{2})^2 - 13\frac{1}{4} = 0$ ,  
 $x = 6.14$  or  $x = -1.14$
43.  $\frac{3x^2+8x}{x^2-9}$
44.  $-\frac{6}{5}$  or 2
45.  $x = \frac{16}{17}$
46. 0.245, -1.18
47.  $4(x-1)^3 - 9$

(b) Find

- (i) the length of  $EG$ ,
- (ii)  $\frac{\text{area of } \triangle DHC}{\text{area of } \triangle DEH}$ .



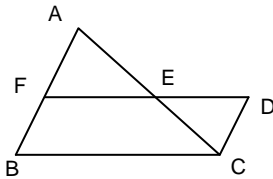
7. In the diagram,  $PQR$ ,  $STR$  and  $PSQ$  are right angled triangles.



- (a) Show that  $\triangle QTS$  is similar to  $\triangle STR$ .  
Given that  $QT = 3$  cm and  $ST = 4$  cm,
- (b) Find  $TR$ .
- (c) Explain why  $ST$  is parallel to  $PQ$ . Hence find  $PQ$ .
- (d) Write the ratio of the area of  $\triangle STR$  to the area of  $\triangle QTS$ .



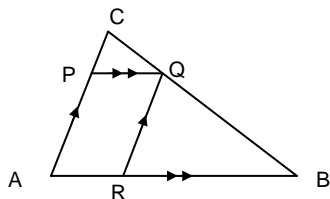
8.



In the diagram,  $AFB$ ,  $AEC$  and  $DEF$  are straight lines.  $FE = DE$  and  $\angle FAE = \angle DCE$ . Show that  $\triangle AFE$  is congruent to  $\triangle CDE$  by stating the reasons clearly.



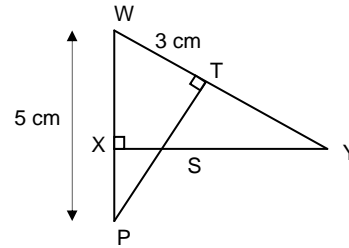
9. In the diagram below,  $APC$ ,  $ARB$  and  $BQC$  are straight lines.  $APQR$  is a parallelogram.



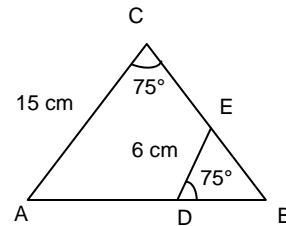
- (a) Show that  $\triangle CPQ$  is similar to  $\triangle QRB$ .
- (b) Given that  $AP : PC = 3 : 1$ ,  $BR = 18$  cm and the area of  $\triangle CPQ$  is  $12$  cm<sup>2</sup>. Find,
  - (i) the length of  $PQ$ .
  - (ii) the area of  $\triangle QRB$ .



10. (a) In the diagram,  $WP = 5$  cm,  $XP = 2$  cm and  $WT = 3$  cm.



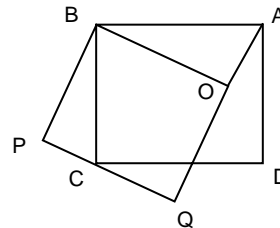
- (i) Show with reasons, that  $\triangle WXY$  is congruent to  $\triangle WTP$ .
  - (ii) Find the length of  $TY$ .
- (b) In the figure below,  $AC = 15$  cm,  $DE = 6$  cm,  $BE = 8$  cm and  $\angle ACB = \angle EDB = 75^\circ$



- (i) Show with reasons, that  $\triangle ACB$  and  $\triangle EDB$
- (ii) Find the length of  $AB$ .
- (iii) If the area of  $\triangle ACB$  is  $200$  cm<sup>2</sup>, calculate the area of the quadrilateral  $ADEC$ .



11. In the diagram below,  $ABCD$  is a square and  $OBPQ$  is a rectangle.  $PCQ$  and  $QOA$  are straight lines.



- (a) Show that angle  $PBC$  is equal to angle  $OBA$ .
- (b) Prove that triangle  $PBC$  is congruent to triangle  $OBA$ .



**Answer keys:****7 • 2****Questions**

1.  $\triangle ADF \equiv \triangle CBE$  (AAS)  
 $\triangle ABE \equiv \triangle CDF$  (RHS or AAS)
3. (a) Triangle AED is similar to Triangle ABC.  
 (b)  $EC = 4.5$  cm  
 (c)  $\frac{49}{32}$
5. (a) (i)  $\frac{5}{3}$   
 (ii)  $50\frac{2}{25}$   
 (b) (ii)  $13\frac{1}{2}$  cm  
 (iii)  $\frac{4}{9}$
6. (b) (i) 3  
 (ii)  $\frac{5}{12}$
7. (b)  $5\frac{1}{3}$   
 (c)  $6\frac{1}{4}$   
 (d) 16 : 9
9. (b) (i) 6 cm  
 (ii) 108 cm<sup>2</sup>
10. (a) (ii) 2 cm  
 (b) (ii) 20 cm  
 (iii) 168 cm<sup>2</sup>
12. (b) (i) 5 cm  
 (ii) 3
13. (a)  $AEB$  and  $CED$   
 (b) 3 : 4
14. (a) (ii)  $17\frac{1}{3}$  cm  
 (iii)  $\frac{9}{49}, \frac{9}{14}$
15. (a)  $ACB$  and  $ADE$   
 (b) 6 cm  
 (c) (i) 9 cm<sup>2</sup>  
 (ii) 10 cm<sup>2</sup>
16. (a)  $ABX$  and  $PDX$   
 (b) 4 : 3
- (c)  $\frac{16}{9}$   
 (d) 4 : 3
18. (a) 5.7  
 (b) 87.7°  
 (c) 20.5 cm<sup>2</sup>
20. (b) (i) 9 : 4  
 (ii) SAS
21. (b) 18.75 cm  
 (c)  $\frac{9}{16}$
22. (a) (ii)  $\frac{2}{5}$   
 (iii)  $\frac{14}{25}$
23. (b) 17
24. (b) (i) 9 cm  
 (ii) 3 cm  
 (iii)  $\frac{9}{4}$   
 (iv)  $\frac{4}{5}$
25. (b)  $\frac{25}{21}$
26. (b) 13.0 cm
27. (a) 150°
28. AAS
29. (a)  $\triangle ORQ, \triangle PRQ$   
 (b) 2 or 8
30. (a) AAA  
 (b) 4 cm  
 (c) 9 : 8
31. (b) (i) 12 cm  
 (ii) 3 : 4
32. SAS
33. (b) 8 cm  
 (c) (i)  $\frac{1}{9}$   
 (ii)  $\frac{1}{18}$
34. By similar volume based on dimensions, consumption has increased by 64 times, instead of the reported 4 times.
35. (b) 2 cm
36. (b)  $\frac{2}{3}$
37. AAS
38. (a)  $\triangle ABE, \triangle ACD$   
 $\triangle BEF, \triangle DCF$   
 (b) 10.5 cm  
 (c) 8.1 cm  
 (d)  $\frac{4}{25}$   
 (e)  $\frac{2}{5}$
39. (a)  $\frac{9}{16}A$   
 (b) 640 cm<sup>3</sup>
40. (a) AAA  
 (b) 10 cm
41. (a) 16.7 cm<sup>2</sup>  
 (b)  $\frac{4}{9}$   
 (c) 16 : 25  
 (d) 3 : 1
42. \$32
43. (f) (i)  $\frac{1}{3}$   
 (ii)  $\frac{1}{12}$
44. (a) ASA  
 (b) 2 cm
45. (a) SSS  
 (b) 0.700 m<sup>2</sup>
46. (b)  $AC = 28$  cm  
 $BQ = 20$  cm  
 (c)  $\triangle ABC = 735$  cm<sup>2</sup>  
 $\triangle PCQ = 360$  cm<sup>2</sup>
47. (b) 4 units<sup>2</sup>
48. (b) AAS
49. (a)  $7776\pi$  cm<sup>3</sup>  
 (b)  $\frac{r_p}{r_s} = \frac{1}{3}$
50. (c)  $\frac{1296}{15625}$
51. (a) AAS  
 (b)  $x = 3.6$   
 $y = 5.2$
52. (a)  $101\frac{1}{4}$  cm<sup>2</sup>  
 (b) 1215 cm<sup>3</sup>