Example 6

The expression $x^2 + x - 3$ has the same $\boxed{\text{remainder}}$ whether divided by $x - a$ or $x + b$, where $a \neq b$. Find the value of $a - b$.

Solution:

Let $f(x) = x^2 + x - 3$, therefore $f(a) = a^2 + a - 3$ and $f(-b) = b^2 - b - 3$.

Since $f(a)$ and $f(-b)$ leave the same remainder,

$$\begin{aligned} f(a) &= f(-b) \\ a^2 + a - 3 &= b^2 - b - 3 \\ a^2 - b^2 + a + b &= 0 \\ (a-b)(a+b) + (a+b) &= 0 \\ (a+b)\left[(a-b)+1\right] &= 0 \\ \text{Since } a \neq b \text{, then } a - b &= -1 \end{aligned}$$

Example 7

One of the solutions of the simultaneous equations given is $(1, a)$.

$$5x + 4y = 13$$

$$12x^2 - 4xy = by^2$$

Find the values of a and b. Hence, find the other solution for the equations.

Solution:

$$5x + 4y = 13 \quad \text{.........} \ (1)$$

$$12x^2 - 4xy = by^2 \quad \text{.........} \ (2)$$

Sub $(1, a)$ into (1) and (2):

(1): $5 + 4a = 13$ .... (3)

(2): $12 - 4a = ba^2$ .... (4)

(3): $a = 2$

(4): $b = 1$

(1): $x = \dfrac{13 - 4y}{5}$ .... (5)